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EXPERIMENTS ON THE INDUCTION OF CURRENTS IN CYLINDRICAL CORES.

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Introduction and Object.

CONSIDER two simple electric circuits of mutual inductance M, one, the primary, having resistance R_1 and inductance L_1, the other, the secondary, having resistance R_2 and inductance L_2. If a harmonic electro-motive force of frequency $f = p/2\pi$ is impressed on the primary and the secondary is closed, the secondary current will react on the primary circuit, changing its effective resistance and inductance from R_1 and L_1 to [1]

$$R' = R_1 + \frac{R_2 M^2 p^2}{R_2^2 + L_2^2 p^2} \quad \text{and} \quad L' = L_1 - \frac{L_2 M^2 p^2}{R_2^2 + L_2^2 p^2}. \qquad (1)$$

Equations (1) are derived from the equation

$$E = I\left[L_1 ip + R_1 + \frac{M^2 p^2 (R_2 - L_2 ip)}{R_2^2 + L_2^2 p^2}\right],$$

where E and I are either the maximum or the effective values of the impressed electro-motive force and the primary current and $i = \sqrt{-1}$.

The impedance of the primary circuit becomes

$$Z = \frac{E}{I} = L_1 ip + R_1 + \frac{M^2 p^2 (R_2 - L_2 ip)}{R_2^2 + L_2^2 p^2}, \qquad (2)$$

where the real part,

[1] See Webster, Electricity and Magnetism, p. 481.

$$R_1 + \frac{R_2 M^2 p^2}{R_2^2 + L_2^2 p^2},$$

is the effective resistance and the imaginary part,

$$ip\left(L_1 - \frac{L_2 M^2 p^2}{R_2^2 + L_2^2 p^2}\right),$$

is ip times the effective inductance.

If the primary circuit is a uniformly wound coil, with straight axis and circular cross-section, and the secondary is a circular cylinder of metal or other conducting material, inside of and coaxial with the coil and of the same length, the reactions of the secondary currents will be of the same character as in the case of two simple circuits. If the coil is so long in comparison to the radius of its section that the magnetic field along any line parallel to the axis may be regarded as uniform throughout its length, then each tube of infinitesimal thickness in the core, co-axial with the coil, will be the seat of a current sheet flowing around it, and producing a magnetic field in its interior. Every current element will flow in a plane normal to the axis. The magnetic field in any tube will be that due to the resultant effect of the coil current and all core currents in tubes outside it. Both the magnetic field and the current will be uniform over any tube, but will vary from tube to tube in intensity and phase, that is, they will be functions of only one geometrical parameter, the distance of the tube from the axis.

If a harmonic electro-motive force is impressed on such a coil, the core current reactions will oppose the setting up of the coil current for a time, but very soon a steady state will be reached in which there will be a harmonic current in the coil and a harmonic variation of strength of current and magnetic field in each tube of the core. These tubes are infinite in number and the system is one with an infinite number of degrees of freedom.

The reactions of the core currents will change the effective resistance and inductance and the impedance of the coil. The object of this work is to study theoretically and experimentally the effect of such reactions.

THEORY.

The following notation is used;

E = electro-motive force impressed on the coil.

H = intensity of the magnetic field.
I = coil current.
q = density of core current.
ρ = resistivity of the core.
μ = permeability of the core.
r = distance from the axis of the coil.
R = resistance of the coil.
c = radius of the core.
L_0 = inductance of the coil = $L_1 + L_2$ where
L_1 = inductance due to the space occupied by the core and
L_2 = inductance due to the space outside the core.
l = length of the coil and the core.
n = number of turns of the coil per centimeter length.
Z = impedance of the coil.
f = frequency in cycles per second of the electro-motive force impressed on the coil.
$p = 2\pi f$.

A tube of thickness dr has flowing in it a current qdr per centimeter of length. This produces inside the tube the magnetic field $4\pi qdr$. Just outside the tube H is less by the part of H within that is due to the current in the tube, or the decrease in H in the radial distance dr is

$$-dH = 4\pi qdr. \tag{3}$$

If e is the electric force in the tube, the electro-motive per centimeter circumferentially, then by Ohm's law

$$e = \rho q, \tag{4}$$

e being entirely due to the change in the magnetic induction within the tube.

From (3) and (4) we have

$$e = -\frac{\rho}{4\pi}\frac{dH}{dr}. \tag{5}$$

The total flux of induction within a tube of radius r is

$$\int_0^r \mu H 2\pi r dr.$$

The time rate of decrease of this flux is equal to the electro-motive

force around the tube, or to the line integral of e around it, that is,

$$2\pi re = -\frac{d}{dt}\int_0^r \mu H 2\pi r dr.$$

From this equation and (5) we have

$$e = -\frac{\rho}{4\pi}\frac{dH}{dr} = -\frac{1}{2\pi r}\frac{d}{dt}\int_0^r \mu H 2\pi r dr.$$

Lord Rayleigh has shown[1] that if the magnetic field intensity is very small, μ is nearly constant for iron, and μ is of course constant for any non-magnetic core. If we assume μ as constant then

$$\frac{\rho}{4\pi}\frac{dH}{dr} = \frac{\mu}{r}\int_0^r \frac{dH}{dt} r dr,$$

or

$$r\frac{dH}{dr} = \frac{4\pi\mu}{\rho}\int_0^r \frac{dH}{dt} r dr.$$

Differentiating and dividing both sides by r we have

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dH}{dr}\right) = \frac{4\pi\mu}{\rho}\frac{dH}{dt}. \qquad (6)$$

If the electro-motive force impressed on the coil is harmonic then H is harmonic at every point and we may put

$$H = e^{pit}U$$

where U is a function of only the geometrical parameter r, and hence

$$\frac{dH}{dt} = ipH = ipe^{pit}U.$$

Also since

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dH}{dr}\right)$$

involves operations by r only, we have

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dH}{dr}\right) = e^{pit}\frac{1}{r}\frac{d}{dr}\left(r\frac{dU}{dr}\right)$$

and thus

[1] See Scientific Papers, Vol. II., page 579.

$$\frac{4\pi\mu}{\rho} ip e^{pit} U = e^{pit} \frac{1}{r} \frac{d}{dr}\left(r \frac{dU}{dr}\right)$$

or

$$\frac{1}{r} \frac{d}{dr}\left(r \frac{dU}{dr}\right) = \frac{4\pi\mu}{\rho} ipU = -k^2 U, \tag{7}$$

where

$$-k^2 = \frac{4\pi\mu ip}{\rho} = ix, \quad \text{putting} \quad x = \frac{4\pi\mu p}{\rho}.$$

The equation

$$\frac{1}{r} \frac{d}{dr}\left(r \frac{dU}{dr}\right) = -k^2 U$$

is satisfied by putting for U the Bessel's function

$$U = CJ_0(kr) = C\left(1 - \frac{k^2 r^2}{2^2} + \frac{k^4 r^4}{2^2 4^2} - \frac{k^6 r^6}{2^2 4^2 6^2} + \cdots\right), \tag{8}$$

where C is any constant. This value of U also satisfies the physical condition of being uniform, finite and continuous over the core, from $r = 0$ to $r = c$, and is accordingly the solution desired.

We have

$$k = \sqrt{-ix},$$

hence

$$J_0(kr) = J_0(r\sqrt{-ix}),$$

$$= 1 + \frac{ixr^2}{2^2} - \frac{x^2 r^4}{2^2 4^2} - \frac{ix^3 r^6}{2^2 4^2 6^2} + \frac{x^4 r^8}{2^2 4^2 6^2 8^2} + \frac{ix^5 r^{10}}{2^2 4^2 6^2 8^2 10^2} \cdots = M(r) + iN(r),$$

where

$$M(r) = 1 - \frac{x^2 r^4}{2^2 4^2} + \frac{x^4 r^8}{2^2 4^2 6^2 8^2} - \frac{x^6 r^{12}}{2^2 4^2 6^2 8^2 10^2 12^2} + \cdots$$

$$N(r) = \frac{xr^2}{2^2} - \frac{x^3 r^6}{2^2 4^2 6^2} + \frac{x^5 r^{10}}{2^2 4^2 6^2 8^2 10^2} - \frac{x^7 r^{14}}{2^2 4^2 6^2 8^2 10^2 12^2 14^2} + \cdots \tag{9}$$

and thus $H = Ce^{pit}\,[M(r) + iN(r)]$.

At the outside of the core $r = c$. Here, and from here out to the inside of the coil, H is uniform and determined by the coil current I. At the surface of the core then

$$H_c = 4\pi nI = Ce^{pit}[M(c) + iN(c)],$$

or

$$I = \frac{Ce^{pit}[M(c) + iN(c)]}{4\pi n}. \tag{10}$$

The inductance L_2 due to the space outside the core is not affected by the core currents. If there are no core currents, the inductance L_1 due to the space filled by the core is $L_1 = 4\pi^2 n^2 c^2 \mu l$. When the core currents flow the number of linkages of turns of the coil with unit tubes of induction in this space is

$$nl\mu \int_0^c H 2\pi r dr,$$

where H is the total field due to the coil and core currents. When E is impressed on the coil a current I will flow. E may at any instant be divided in three parts. The first, RI is used to overcome the ohmic resistance of the coil ; the second,

$$L_2 \frac{dI}{dt} = (L_0 - L_1) \frac{dI}{dt},$$

to overcome the counter electro-motive force due to the change of induction in the space outside the core ; and the third,

$$\frac{d}{dt}\left(nl\mu \int_0^c H 2\pi r dr \right),$$

to overcome the similar counter electro-motive force due to the change of induction in the core space. We have then

$$E = RI + (L_0 - L_1) \frac{dI}{dt} + 2\pi nl\mu \int_0^c \frac{dH}{dt} r dr,$$

or by (6)

$$E = RI + (L_0 - L_1) \frac{dI}{dt} + \frac{\rho nl}{2} \int_0^c \frac{d}{dr}\left(r \frac{dH}{dr} \right) dr$$

$$= RI + (L_0 - L_1) \frac{dI}{dt} + \frac{\rho nlc}{2} \left[\frac{dH}{dr} \right] (r = c). \qquad (11)$$

Now

$$\frac{dH}{dr} = Ce^{pit} \frac{J}{dr} [M(r) + iN(r)].$$

Let us put $M'(r)$ for

$$\frac{dM(r)}{dr} = -\frac{x^2 r^3}{2^2 4} + \frac{x^4 r^7}{2^2 4^2 6^2 8} - \frac{x^6 r^{11}}{2^2 4^2 6^2 8^2 10^2 12} + \cdots;$$

$N'(r)$ for

$$\frac{dN(r)}{dr} = \frac{xr}{2} - \frac{x^3 r^5}{2^2 4^2 6} + \frac{x^5 r^9}{2^2 4^2 6^2 8^2 10} - \frac{x^7 r^{13}}{2^2 4^2 6^2 8^2 10^2 12^2 14} + \cdots;$$

M for

$$M(c) = 1 - \frac{x^2c^4}{2^2 4^2} + \frac{x^4c^8}{2^2 4^2 6^2 8^2} - \frac{x^6c^{12}}{2^2 4^2 6^2 8^2 10^2 12^2} + \cdots;$$

N for

$$N(c) = \frac{xc^2}{2^2} - \frac{x^3c^6}{2^2 4^2 6^2} + \frac{x^5c^{10}}{2^2 4^2 6^2 8^2 10^2} - \frac{x^7c^{14}}{2^2 4^2 6^2 8^2 10^2 12^2 14^2} + \cdots;$$

M' for $M'(r)$ when $r = c$ giving

$$M' = -\frac{x^2c^3}{2^2 4} + \frac{x^4c^7}{2^2 4^2 6^2 8} - \frac{x^6c^{11}}{2^2 4^2 6^2 8^2 10^2 12} + \cdots;$$

N' for $N'(r)$ when $r = c$ giving

$$N' = \frac{xc}{2} - \frac{x^3c^5}{2^2 4^2 6} + \frac{x^5c^9}{2^2 4^2 6^2 8^2 10} - \frac{x^7c^{13}}{2^2 4^2 6^2 8^2 10^2 12^2 14} + \cdots.$$

We get then from (11)

$$E = RI + (L_0 - L_1)\frac{dI}{dt} + \frac{\rho nlc}{2} Ce^{pit}(M' + iN'). \qquad (12)$$

Since now E is harmonic it may be represented by $E = Ae^{pit}$, and I will be harmonic and may be represented by $I = Be^{pit}$, or as in (10) by

$$\frac{Ce^{pit}(M + iN)}{4\pi n} \quad \text{and} \quad \frac{dI}{dt} = ipBe^{pit} = ipI.$$

The impedance of the coil is $Z = E/I$ and we get

$$Z = \frac{RI}{I} + \frac{(L_0 - L_1)iPI}{I} + \frac{\rho nlc\, Ce^{pit}(M' + iN')4\pi n}{2Ce^{pit}(M + iN)}$$

$$= R + (L_0 - L_1)ip + 2\pi n^2 \rho lc \frac{M' + iN'}{M + iN}$$

$$= R + (L_0 - L_1)ip + \frac{L_1\rho}{2\pi\mu c} \frac{MM' + NN' + i(MN' - M'N)}{M^2 + N^2}. \qquad (13)$$

By analogy with (2) we may put the real part of Z equal to the effective resistance

$$R' = R + \frac{L_1\rho}{2\pi\mu c} \frac{MM' + NN'}{M^2 + N^2}. \qquad (14)$$

and the imaginary part, divided by ip, equal to the effective inductance

$$L' = L_0 + L_1 + \frac{L_1\rho}{2\pi\mu c p}\frac{MN' - M'N}{M^2 + N^2}$$
$$= L_0 - L_1\left(1 - \frac{\rho}{2\pi\mu c p}\frac{MN' - M'N}{M^2 + N^2}\right). \quad (15)$$

The absolute value of the impedance is

$$Z = (R'^2 + L'^2p^2)^{\frac{1}{2}} = \left[R^2 + (L_0 - L_1)^2p^2 + \left(\frac{L_1\rho}{2\pi\mu c}\right)^2\frac{M'^2 + N'^2}{M^2 + N^2}\right.$$
$$\left. + \frac{R\rho L_1}{\pi\mu c}\frac{MM' + NN'}{M^2 + N^2} + \frac{\rho L_1 p(L_0 - L_1)}{\pi\mu c}\frac{MN' - M'N}{M^2 + N^2}\right]^{\frac{1}{2}}.$$

Heaviside[1] has given results from which this value of Z may readily be obtained.

By (14) and (15) R' and L' are expressed in terms of geometrical, electrical and magnetic constants of the coil and core, and of the four series M, N, M' and N', which are functions of the same constants and of the frequency. The formulas deduced are independent of the length of the coil, provided that the coil and the core are of the same length, and so long in comparison to the radius of the coil section that the magnetic field may be assumed to be uniform along any parallel to the axis without appreciable error.

APPARATUS AND METHOD.

The direct subjects of experimental observation were the effective resistance and inductance of a coil with different metallic cores and at various frequencies of the impressed electro-motive force. From these results the impedance under the same conditions was calculated.

The effective resistance and inductance were also calculated from (14) and (15), and from their values the theoretical impedance was calculated.

A tube was made of paper wound on a form in layers, the inside diameter being 1.905 and the outside diameter 2.54 centimeters. The paper was wound on a $\frac{3}{4}$ inch bar, the layers being stuck together with shellac. Wooden end pieces were glued on, leaving a winding space 38.75 centimeters long. Three layers of number

[1] See Electrical Papers, Vol. I., page 353.

29 double cotton covered wire were wound on, the layers having 625, 624 and 625 turns respectively, giving a density of winding of $n = 1{,}874/38.75 = 48.361$.

The resistance of the coil was carefully measured with a Wheatstone bridge and is $R = 49$ ohms $= 49 \times 10^9$ at 22° C. The inductance was measured by Maxwell's bridge method, using an alternating current and telephone, as described by Max Wien,[1] and was found to be $L = 7.2$ milli-henrys $= 7.2 \times 10^6$. The alternating current was supplied by a small rotary converter run as an inverted rotary from a storage battery, and supplying current at two or three volts. As a standard of inductance for this and the later work a movable coil variable inductance, Ayrton and Perry form, made by the Leeds and Northrup Co. was used. This standard, which had a range from 4 to 40 milli-henrys, had been calibrated previously by Miss Agnes Childs, by comparing it with a coil wound on plaster whose inductance had been calculated from Maxwell's formula.

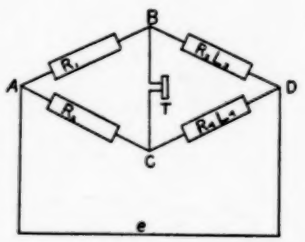

Fig. 1.

In measuring the resistance and inductance of the coil with the core inserted, the connection was as in Fig. 1. R_1 and R_2 are non-inductive resistances, in the experiments the 100-ohm and 20-ohm coils of a box of coils were used. R_3L_3 is the movable coil standard inductance, with non-inductive resistance added, R_4L_4 is the coil under examination. In the work R_1 and R_2 were taken from a box of coils, the ratios of the coils of which had been adjusted with especial care by Professor Hooper of Tufts College. This was important because the ratio comes into the results instead of the actual value of either. R_3 is the resistance of the standard inductance, 9.25 ohms, in series with as much as was needed to give a balance, taken from a Leeds box. When a harmonic difference of potential is maintained between the points A and D there will be silence in the telephone T if the resistances and inductances have the relations $L_4/L_3 = R_4/R_3 = R_2/R_1 = .2$, since R_2 and R_1 were kept at 20 ohms and 100 ohms respectively and the adjustment for balance made entirely in R_3 and L_3.

[1] See Annalen der Physik und Chemie, Bd. 44, 1891, pp. 681–688.

The reaction of the core currents on the resistance and inductance of the coil varies with the frequency, see (1), accordingly there is no adjustment of resistances and inductances which will produce a balance and hence silence in the telephone unless the electro-motive force is harmonic. It was at first thought that this fact would make it impossible to use a telephone, as no means were available to produce a harmonic wave of electro-motive force of sufficient intensity and range of frequency for the work.

The first attempt therefore was to use a vibration galvanometer constructed on the principle of the one described by Max Wien,[1] with which he had secured good results.

With an instrument of this sort and for the purpose in mind the electro-motive force need not be even approximately harmonic for successful working, the one essential is that the fundamental period shall remain constant. The effect of the components of higher frequency than the one for which the instrument is adjusted is hardly noticeable. A tuning fork can be arranged to open and close the primary circuit of an induction coil with the required regularity, thus giving an alternating current in the secondary, but the frequency which can be obtained in this way is comparatively low, and its range is small, while the range that is obtainable requires a set of rated forks or else considerable labor in the rating of an adjustable fork at different speeds. On this account a great deal of work was done and much time was spent in trying to run an alternating current generator steadily enough to be used as the source of supply of the current. In the first place a 56 pole field magnet was mounted on the shaft of a direct current motor driven from a storage battery, an alternating current armature being supplied in proper relation to the field magnet. This attempt was not successful, as the speed varied between altogether too wide limits. A small direct current motor was supplied with collector rings and connected as a rotary converter, to be run as an inverted rotary from a storage battery, the same in fact as was used in measuring the inductance of the coil. Several governors were tried, but none could be devised that would give the required steadiness of speed, and it was finally concluded that the measurements could not be

[1] See Annalen der Physik, Bd. 4, 1901, pp. 440–445.

made with the galvanometer at any but low frequencies. This plan was therefore dropped, as the core reactions increase rapidly with the frequency and hence measurements at high frequencies were especially desired. The writer has given[1] an account in an earlier paper of the results of certain experiments with the Wien vibration galvanometer.

The next attempt, and one that proved successful, was with a telephone. The first plan was to take the current used in the measurements from the secondary winding of an air core transformer. Capacity was to be inserted in series with the primary, so adjusted as to counterbalance the inductance in the primary and dynamo for the fundamental frequency, and thus make the overtones comparatively small in amplitude, obtaining an approximately harmonic current in the secondary and in the observing apparatus, as described by Pupin.[2]

As a source of primary current a motor-generator set in the electrical engineering laboratory of Tufts College was selected. The alternating current generator of this set was designed and partly built by the writer about ten years ago while a student in Tufts College. The set was intended for experimental purposes, and it, together with any other of his apparatus desired, was put at the disposal of the writer, for this work, by Professor Hooper.

The generator is of the Mordey disk type, with 24 pairs of poles and 48 armature coils. The armature is divided into twelve sections of four coils each, with connections brought out to a switchboard, and so arranged that the sections can be put into various combinations of series and multiple grouping. The shaft is rigidly clutched to the shaft of the driving motor of the set, a six pole, shunt wound, continuous current motor, run from a storage battery. The set will run steadily at any speed from 150 to 2,600 revolutions per minute, giving a range of frequency from 60 to 1,040 cycles per second.

The form of the electro-motive force wave is not exactly known, but it is not far from harmonic, enough however so that the telephone gave a distinct sound for any adjustment of the bridge arms with a core in the coil. Preliminary experiments were made with

[1] PHYSICAL REVIEW, Vol. XXIII., No. 6, December, 1906, p. 504.

[2] American Journal of Science, Vol. 48, 1894, pp. 379–389 and 473–485.

the current from a 16 pole alternator run from a steam engine, and the attempt was made to balance with capacity the inductance of the primary of an air core transformer and the dynamo in series. In getting the circuit into electrical resonance a dynamometer was inserted in the primary and the capacity adjusted to give the maximum current, which was considered to be the point of resonance. With the condensers available it was in general possible to get only approximate resonance, and this was frequently annuled by the change in the inductance of the primary on closing the secondary. A few attempts were then made to take bridge measurements without any attempt to weed out the overtones of the current wave. It was found that with the shrill sound of the overtones always present in the telephone it was easy to fix a balance for the fundamental frequency, and this with great accuracy for frequencies of 250 cycles per second or over. With lower frequencies the possible accuracy of adjustment is a little less, and accordingly readings were taken at less intervals of frequency at low frequencies than at high, in order to get sufficient data for plotting smooth curves of resistance and inductance.

RESULTS.

On account of the varying permeability of iron it was decided to confine attention to non-magnetic cores, and copper, aluminum and brass were selected as giving a good range of resistivities. Cores of each metal were prepared, 38.75 centimeters long and 1.905 centimeters in diameter. The brass was cut from a stock bar in the laboratory shop and its resistivity measured. As a standard of resistance, a piece of soft drawn trolley wire was especially drawn by the American Steel and Wire Co., who also furnished its resistance. This was put in series with the brass, and a current passed through the two. A frame was prepared carrying two knife edges insulated from each other and 35 centimeters apart, which were laid on the bar. Two knife edges were prepared which could be put in contact with the trolley wire at a variable measured distance apart. Terminals from all four knife edges were brought to a Pohl's commutator, by which either set could be connected to the galvanometer, and a reversing switch was inserted in the circuit. By adjusting the second pair of knife edges to the proper distance apart, the

difference of potential between edges could be made the same in the two pairs, as shown by equal galvanometer deflection. Since the current in the brass bar was the same as that in the copper wire the resistances between the pairs of connected points was the same, and the resistivity of the brass could be obtained in terms of that of the copper. Four comparisons were made, the deflections being equalized in both directions for each of two current values. The four results agreed very well, giving as a mean the value of the resistivity of the brass as $\rho_b = 7{,}850$. The only galvanometer available was of the needle type, and although it was surrounded by a thick shield of iron filings, it was a good deal affected by the magnetic field from a nearby car line. In measuring the resistivity of the aluminum bar the maximum deflection that could be obtained was small on account of the low resistance of the bar and the drift due to the car line was serious. The value obtained is $\rho_a = 3{,}035$, which is liable to an error of a few per cent. due to the drift. On account of the cer-

TABLE I.

f	Copper.		Aluminum.		Brass.	
	R'	L'	R'	L'	R'	L'
0	49.00	7.20	49.00	7.20	49.00	7.20
78	49.65	6.20	49.65	6.75	49.85	6.86
100	49.65	6.46	49.25	6.80	49.95	7.00
124	50.05	6.24	49.85	6.75	49.45	7.14
145	50.35	6.17	50.35	6.48	49.85	7.06
160	50.45	6.02	50.35	6.45	49.75	7.04
188	50.35	6.17	50.35	6.48	49.85	7.06
196	50.75	5.68	50.75	6.28	49.95	6.97
205	50.85	5.64	50.85	6.30	50.35	6.94
225	50.95	5.46	51.05	6.10	50.35	6.90
234	50.75	5.47	50.85	6.07	50.25	6.90
277	51.25	5.30	51.55	5.85	50.95	6.76
348	51.45	5.10	51.95	5.56	51.75	6.57
429	51.95	4.96	52.55	5.35	52.65	6.35
490	52.25	4.92	53.05	5.24	53.45	6.17
556	51.95	4.84	52.85	5.12	53.55	6.05
640	52.85	4.78	53.65	5.04	54.75	5.84
732	53.05	4.74	54.05	4.95	55.45	5.69
806	53.55	4.68	54.35	4.89	55.95	5.55
811	53.05	4.69	54.05	4.84	55.65	5.56
938	53.55	4.64	54.65	4.80	56.65	5.40
1,032	53.75	4.59	54.95	4.78	57.25	5.35

TABLE II.

f	Copper.		Aluminum.		Brass.	
	R'	L'	R'	L'	R'	L'
100	49.71	6.46	49.52	6.88	49.23	7.15
200	50.57	5.69	50.48	6.29	49.85	7.00
300	51.00	5.31	51.30	5.83	50.73	6.79
400	51.38	5.11	51.90	5.35	51.71	6.55
500	51.71	4.98	52.35	5.33	52.68	6.33
600	52.01	4.89	52.76	5.19	53.58	6.12
700	52.41	4.82	53.12	5.09	54.34	5.94
800	52.55	4.76	53.45	5.02	55.11	5.80
900	52.74	4.71	53.76	4.95	55.75	5.67
1,000	52.78	4.67	53.94	4.90	56.59	5.60

tainty of a still greater error in the measurement of the resistivity of the copper, because its lower value would cause a still smaller galvanometer deflection, Matthiessen's value of 1,590 at 0° C., and a temperature coefficient of .4 per cent. were used, and 3 per cent. added for impurities, giving a value $\rho_c = 1{,}782$ at 22° C.

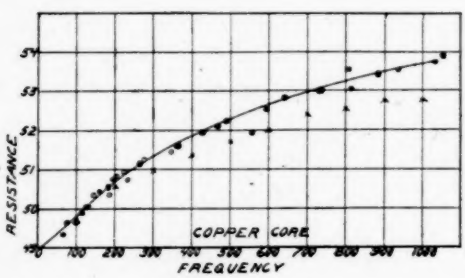

Fig. 2.

In taking the final observations with coil and core an incandescent lamp was inserted in the dynamo circuit, to make sure that no injuriously large current should flow through the box of coils.

The observed resistances and inductances of the coil with the different cores and at different frequencies are given in Table I., R' being in ohms and L' in milli-henrys.

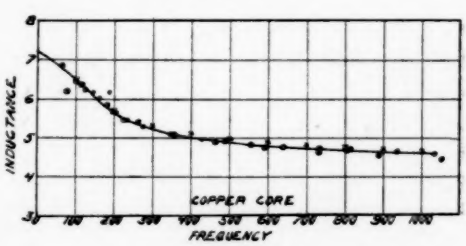

Fig. 3.

The values of R' and L' as calculated from (14) and (15) are given in Table II., for the three cores, at frequencies up to 1,000 cycles per second, by steps of 100 cycles, the value

of L_1 used being given by the expression

$$L_1 = 4\pi^2 n^2 c^2 l = 4\pi^2 \times 48.361^2 \times .9525^2 \times 38.75 = 3.246 \times 10^6.$$

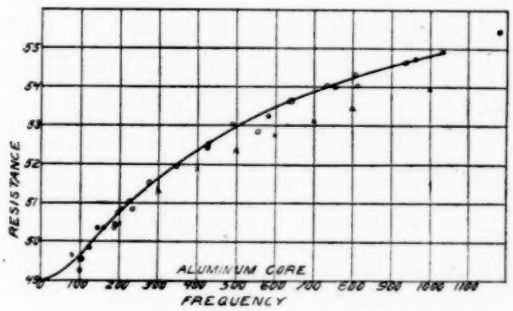

Fig. 4.

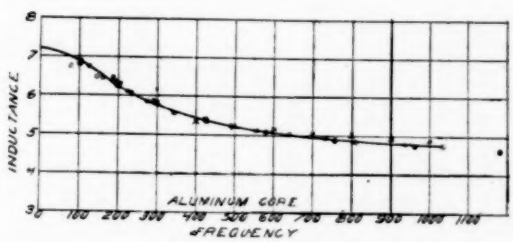

Fig. 5.

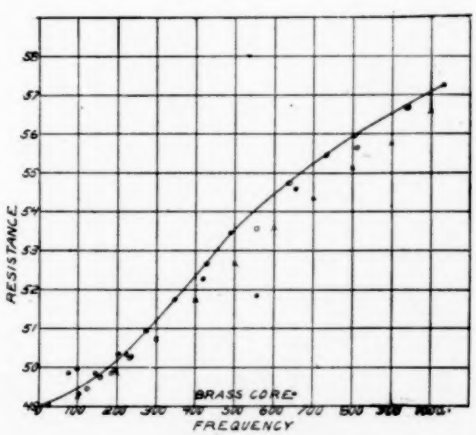

Fig. 6.

The values in Tables I. and II. are plotted in Figs. 2, 3, 4, 5, 6 and 7. In these curves the observed points are shown as dots sur-

rounded by circles, a full line being drawn, while the points calculated from (14) and (15) are shown as crosses.

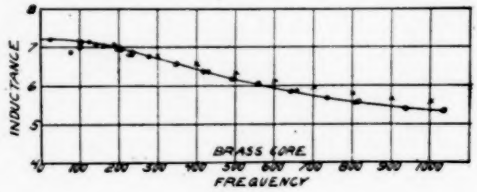

Fig. 7.

The impedance of the coil with the three cores was calculated for various frequencies up to 1,000, from the curves of observed values of R' and L' by steps of 50, and from the values of R' and L' from (14) and (15) by steps of 100. These results are given in Table III.

These results are plotted in Figs. 8, 9 and 10 using the same convention as to circles, crosses and kinds of lines as in Figs. 2 to 7.

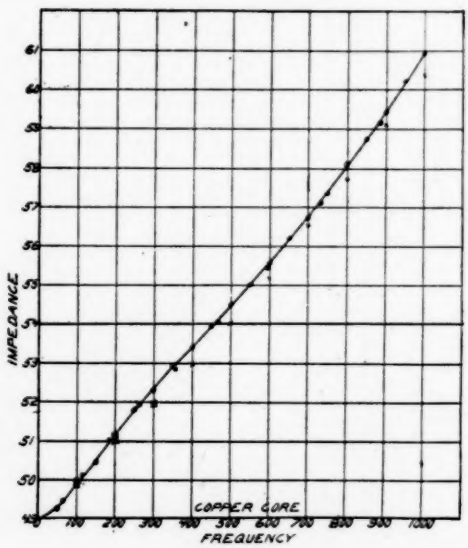

Fig. 8.

Table IV. gives the difference in the increase of impedance with frequency, which is the real test of the accuracy of the work, as obtained from formulas (14) and (15), and from the observations, marked + or — according as the latter is larger or smaller than the

former, and the percentage that this difference is of the value as obtained from the formulas.

TABLE III.

f	Copper.		Aluminum.		Brass.	
	Formulas.	Observed.	Formulas.	Observed.	Formulas.	Observed.
0		49.00		49.00		49.00
50		49.25		49.14		49.05
100	49.88	49.94	49.71	49.70	49.43	49.40
150		50.48		50.58		49.97
200	51.07	51.19	51.10	51.42	50.62	50.76
250		51.80		52.13		51.62
300	51.98	52.32	52.46	52.78	52.32	52.62
350		52.92		53.30		53.61
400	52.96	53.40	53.73	54.08	54.27	54.68
450		53.96		54.73		55.76
500	54.02	54.50	54.97	55.39	56.31	56.75
550		55.02		56.13		57.74
600	55.18	55.56	56.27	56.72	58.34	58.80
650		56.22		57.42		59.73
700	56.56	56.75	57.65	58.08	60.29	60.65
750		57.38		58.81		61.68
800	57.74	58.11	59.11	59.46	62.35	62.47
850		58.72		60.19		63.40
900	59.10	59.42	60.61	60.98	64.31	64.36
950		60.22		61.73		65.21
1,000	60.39	60.95	62.11	62.51	66.64	66.14

TABLE IV.

f	Copper.		Aluminum.		Brass.	
	Diff.	Per Cent. Diff.	Diff.	Per Cent. Diff.	Diff.	Per Cent. Diff.
100	+.06	6.82	−.01	1.41	−.03	6.98
200	+.12	5.80	+.32	15.24	+.14	8.64
300	+.34	11.41	+.32	9.25	+.30	9.04
400	+.44	11.11	+.35	7.40	+.41	7.78
500	+.48	9.56	+.42	7.04	+.44	6.02
600	+.38	6.15	+.45	6.19	+.46	4.93
700	+.19	2.51	+.43	4.97	+.36	3.19
800	+.37	4.23	+.35	3.46	+.12	.90
900	+.32	3.17	+.37	3.19	+.05	.33
1,000	+.56	4.92	+.40	3.05	−.50	2.83
	Mean 6.57%		Mean 6.12%		Mean 5.06%	

An examination of the tables and curves shows that in nearly every case the observed resistance is more and the observed inductance is less than the results calculated from (14) and (15). In general, however, the two sets of values lie on curves of the same shape, and near enough together so that we may consider the

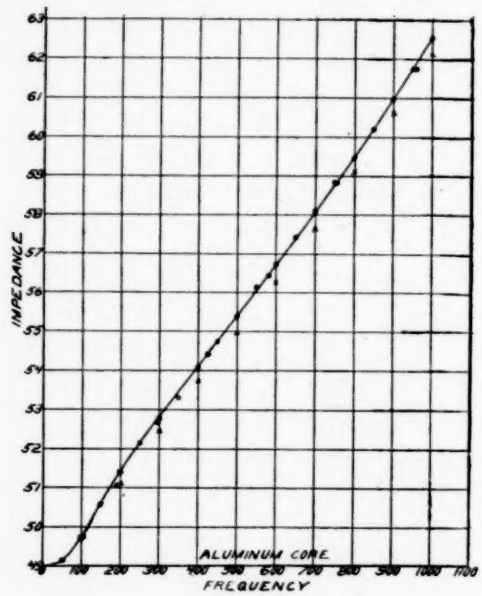

Fig. 9.

formulas deduced to be verified by experiment. The differences are undoubtedly due, partly to errors of observation, and to greater degree to using in the calculations incorrect values of the resistivity and the inductance of the core. In the impedance curves the agreement is much better, the differences in resistance and inductance tending to compensate for each other.

The curves of Figs. 2 to 7 show that both resistance and inductance tend to a limiting value as the frequency increases, the frequency at which the limiting value of either is reached increasing with the resistivity of the core. The final limits of resistance and inductance seem to increase with the resistivity of the core also, as would be expected from (1).

For comparison the impedance of the coil with no core and the observed impedances with the different cores are given in Table V.

TABLE V.

f	Copper.	Aluminum.	Brass.	No Core.
0	49.00	49.00	49.00	49.00
100	49.88	49.71	49.43	49.21
200	51.07	51.10	50.62	49.83
300	51.98	52.46	52.32	50.84
400	52.96	53.73	54.27	52.23
500	54.02	54.97	56.31	53.97
600	55.18	56.27	58.34	56.02
700	56.56	57.65	60.29	58.34
800	57.74	59.11	62.35	60.92
900	59.10	60.61	64.31	63.71
1,000	60.39	62.11	66.64	66.69

These results are plotted in Fig. 11. From them it is seen that at low frequencies the impedance is increased by the insertion of any one of the cores, but as the frequency increases the decrease in inductance when multiplied by 2π times the frequency is more than enough to counterbalance the increase in resistance, and the impedance is decreased by the insertion of a core. Thus the curve of impedance with no core crosses successively the curves of impedance with the different cores, and at a frequency increasing with the increase in resistance change and decrease in inductance change due to the core, that is, with the resistivity of the core.

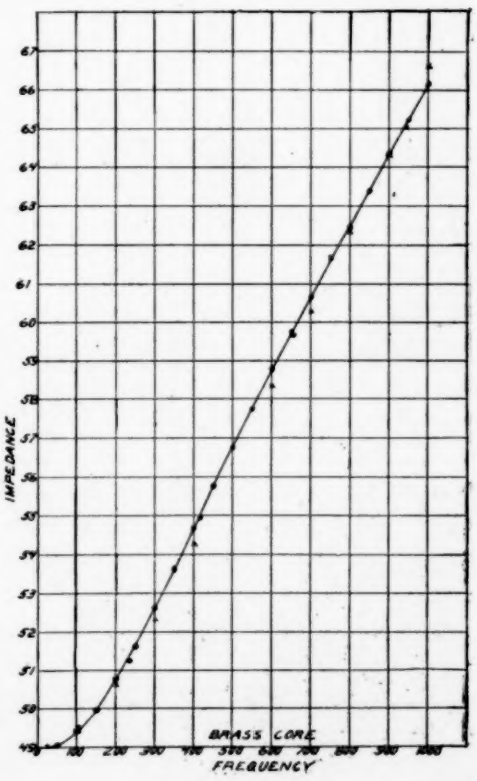

Fig. 10.

The results obtained from observation and those obtained by computation from formulas (14) and (15) agree in the shape of the curves in general, there is, however, a considerable difference, as shown by Table IV., the difference in increase of impedance being from .33 per cent. to 15.24 per cent., with a mean of 5.92 per cent. While the similarity in the shapes of the curves proves definitely the correctness of the theory as developed, the fact that the computed resistance is too great and the computed inductance too small, in nearly every case, indicates that a systematic error was made in the computations.

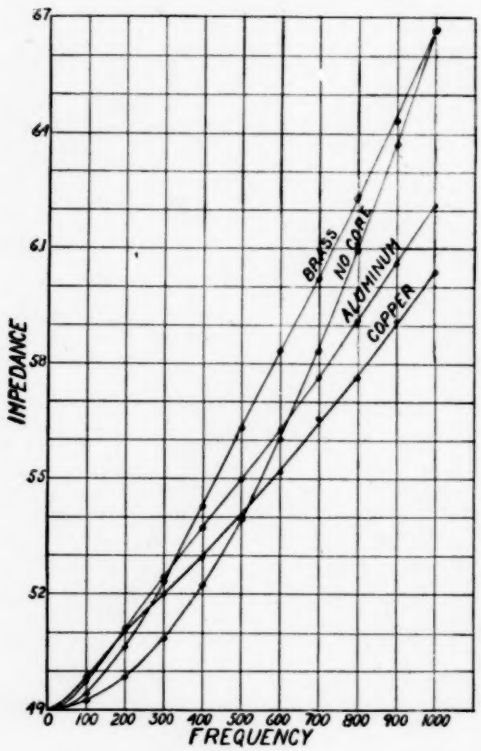

Fig. 11.

The computation of resistance and inductance involves two constants for each core, namely, the resistivity of the core and the inductance L_1, the latter being the same for all cores, and it is probably due to using incorrect values of these constants in the computations that the curves do not agree better. The determination of all of them was subject to error with the means available. Mr. Gordon Fulcher, assistant to Professor A. G. Webster, has at the suggestion of the latter determined by a process of trial values for L_1 and the resistivities of the cores which when used in formulas (14) and (15) give results which agree closely with those observed, and those computed from the observations. These values found by Mr. Fulcher are $L_1 = 3.60 \times 10^6$, and resistivities of 2,100, 3,400 and 7,500 for copper, aluminum and brass respectively. When these values are used in

formulas (14) and (15) they give the following results, resistance and inductance directly and impedance by calculation.

TABLE VI.

Copper.				Aluminum.				Brass.			
	R'	L'	Z	f	R'	L'	Z	f	R'	L'	Z
66	49.34	6.87	49.46	107	49.56	6.87	49.77	26	49.02	7.195	49.03
117	49.91	6.39	50.13	190	50.48	6.39	51.05	105	49.29	7.145	49.51
183	50.56	5.88	51.01	297	51.53	5.88	52.68	236	50.23	6.87	51.25
264	51.15	5.40	51.93	427	52.48	5.40	54.44	419	52.27	6.39	54.91
359	51.64	5.09	52.86	581	53.28	5.09	56.43	654	54.58	5.88	59.69
469	52.10	4.89	54.06	759	54.01	4.89	58.83	942	56.67	5.40	65.06
594	52.55	4.74	55.45	961	54.74	4.74	61.77				
733	53.00	4.62	57.11	1,186	55.48	4.62	65.29				
889	53.47	4.53	59.15								
1,055	53.94	4.45	61.48								

These results are plotted in Figs. 2 to 10, being shown as heavy dots, and show an excellent agreement with the results as obtained from observation.

In conclusion there remains only to express my thanks to all who have assisted in making this work possible; to Mr. Southerland of the American Steel and Wire Co. for having drawn for me the soft copper used, to Mr. Fulcher for his tedious work of computation, to Professor Hooper, of Tufts College, for placing his equipment at my disposal, to Clark University for the opportunity to work in the laboratory, and especially to Professor A. G. Webster for his continual kindness, both in suggesting the work and in assisting while it was in progress.

CLARK UNIVERSITY,
WORCESTER, MASS.,
June, 1903.

INDUCTION COILS.

BY BENJ. F. BAILEY.

MY object in this investigation was first and principally to establish the conditions of maximum efficiency in primary coils, and to deduce a method whereby the efficiency of such coils could be predetermined from their electrical design; secondly to verify experimentally this theory; and thirdly to study the ordinary induction coil with secondary current from both theoretical and experimental standpoints. The efficiency of the secondary coil was also investigated and determined experimentally in the case of one coil.

By an induction coil I mean an apparatus in which magnetic energy is stored, to be used later in some different form from that in which it was supplied. Ordinarily, it is also to be understood that the charging of the coil is to be accomplished from a constant potential source, and usually, the intention is to utilize the stored energy at a higher potential and at a greater wattage than that at which it was supplied. Such coils are made in a great variety of forms. The simplest is a plain coil of one or more turns of a non-magnetic conductor, and an arrangement for making and breaking the circuit. Such an apparatus will not store much energy per unit of weight and consequently can be used economically only when it is charged and discharged a great many times per second. An example of this is the Tesla high frequency coil. For ordinary purposes the introduction of an iron core greatly increases the capacity of the coil. The voltage obtainable from such a coil is still rather limited, and if higher values are desired, they may be obtained by winding a secondary coil of a greater number of turns around the primary coil. For most purposes this arrangement is improved by the use of a condenser connected across the break. These various forms of the apparatus will be taken up in the order described.

Primary or Single Circuit Coils.

The primary coil, while a relatively unimportant device, is used in the ignition of almost all stationary gas engines and with many boat and automobile engines. Moreover, the theory of these coils as developed here will be used directly in considering the case of the double circuit or ordinary induction coil. This, together with the fact that so far as I know, nothing has been written regarding the theory of the primary coil, is the reason for my taking it up here. The first point to be considered is the efficiency of such coils.

Efficiency of Primary Coil.

By the efficiency of any piece of electrical apparatus we mean the ratio of the useful work obtained from it to the total work put into it. In obtaining this for any piece of apparatus two methods are in general available. The most obvious one is to measure these quantities directly. The case of the simple primary circuit is shown in Fig. 1. To apply this method we might use a wattmeter as indicated to measure the input. Theoretically, the output might be measured by transferring the connection *a*, from the terminal of the battery to the other side of the break at *b*. Practically, this would not work well for several reasons. For one thing the high voltage at the break would be liable to puncture the insulation of the wattmeter. Moreover, the amount of power required to operate the wattmeter is large compared to the power of the spark; or stated differently, a large part of the current would shunt off through the wattmeter instead of passing across the gap. This would obviously affect the intensity of the spark, making it much less than it should be. Also since the current and E.M.F. are far out of phase, the self-induction of the wattmeter coils would cause the results to be in error to a considerable extent.

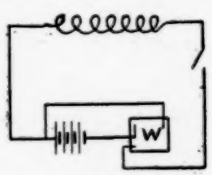

Fig. 1.

Another method of accomplishing the same object would be to measure the heat produced by the spark by means of a small calorimeter. This method is difficult to apply and is not very accurate.

The method here deduced is in principle the same as that used to predetermine the efficiency of all large generators, transformers, etc.

It is based upon the principle of the conservation of energy, *i. e.*, all the energy that goes into the coil must appear in some form or other. Part appears as the energy of the spark and the rest is wasted in various losses. Stated mathematically we have

$$\text{Eff.} = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}.$$

This method is comparatively easy to apply to the case of a dynamo or transformer. With a spark coil it is more difficult, as the calculations are more involved owing to the shape of the waves used. It should be remarked that while the method appears indirect at first, it is in reality preferable to the direct method, both on the score of accuracy, and even more because it shows the magnitude of the various losses, and thus indicates how the design of the coil may be modified so as to improve it.

COIL WITHOUT IRON CORE. INSTANTANEOUS BREAK.

We will first take up the case of a coil without an iron core. This is the simplest case as there are no iron losses to consider. In a circuit like that of Fig. 1 in which the total resistance of the circuit is R and the inductance is L, the differential equation is:

$$E = Ri + L\frac{di}{dt}$$

and

$$i = I(1 - \varepsilon^{-Rt/L}).$$

The magnetic energy stored in the coil at any instant is $\frac{1}{2}Li^2$. If the circuit is broken instantaneously, all of this energy (with the possible exception of a small loss due to eddy currents in the copper) appears as the energy of the spark. This will be practically all heat energy, but a small proportion may be radiated as electromagnetic waves. It is here assumed that all the energy of the spark is used, and the output is consequently $\frac{1}{2}LI^2$.

The input will be the output plus the losses. The latter here consist merely of the ohmic loss in the winding, during the process of building up the magnetism. To get this we must find

$$\int_0^T Ri^2 dt.$$

R is a constant and substituting for i we get

$$R\int_0^T i^2 dt = R\int_0^T I^2\left(1-\varepsilon^{-\frac{Rt}{L}}\right)^2 dt$$

$$= RI^2\int_0^T\left(1-2\varepsilon^{-\frac{Rt}{L}}+\varepsilon^{-\frac{2Rt}{L}}\right)dt.$$

Integrating this we get

$$RI^2\left[T+\frac{L}{2R}\left(1-\varepsilon^{-\frac{2RT}{L}}\right)-\frac{2L}{R}\left(1-\varepsilon^{-\frac{RT}{L}}\right)\right]=Om.$$

The total input is

$$\tfrac{1}{2}Li^2+Om=\tfrac{1}{2}Li^2+RI^2T+\tfrac{1}{2}LI^2(1-\varepsilon^{-2RT/L})-2LIi$$

which readily reduces to RI^2T-LIi. The efficiency then becomes

$$\eta=\frac{\frac{1}{2}Li^2}{RI^2T-LIi}.$$

This gives the efficiency in terms of L, i, R, I, and T. It is obvious that these quantities are interconnected, and by eliminating we should arrive at a simpler form. To eliminate T take logarithms of both sides of

$$\frac{I-i}{I}=\varepsilon^{-\frac{RT}{L}}$$

and get

$$T=\frac{L}{R}\log\frac{I}{I-i}.$$

Substituting this in the above we obtain

$$\eta=\frac{\frac{1}{2}i^2}{I^2\log\frac{I}{I-i}-Ii}.$$

This gives us at once the very peculiar results that the efficiency is independent of both the resistance and the inductance and depends only on the value of the current at the instant of break and upon its steady value. This may be made clearer and some new laws brought out by a further change.

To do this substitute

$$\log\frac{I}{I-i}=\left(\frac{i}{I}\right)+\frac{1}{2}\left(\frac{i}{I}\right)^2+\frac{1}{3}\left(\frac{i}{I}\right)^3+\cdots,$$

simplify and get

$$\eta = \frac{\frac{1}{2}\left(\frac{i}{I}\right)^2}{\frac{1}{2}\left(\frac{i}{I}\right)^2 + \frac{1}{3}\left(\frac{i}{I}\right)^3 + \frac{1}{4}\left(\frac{i}{I}\right)^4 + \cdots}.$$

Or if we designate the ratio i/I by a we obtain

$$\eta = \frac{\frac{1}{2}a^2}{\frac{1}{2}a^2 + \frac{1}{3}a^3 + \frac{1}{4}a^4 + \cdots}.$$

From this very simple form we can at once deduce several important conclusions.

I. The efficiency is entirely independent of the ohmic resistance of the coil, for a given ratio of i/I.

II. The efficiency is entirely independent of the inductance of the coil.

III. The efficiency is a function of the ratio of the current at break, to the current which would flow if the contact were very long.

IV. The efficiency is a maximum and equal to 100 per cent. for $i = 0$.

V. The efficiency is a maximum and equal to 100 per cent. for $I = \infty$.

We may write the expression for the efficiency as follows:

$$\eta = \frac{\frac{1}{2}\left(\frac{iR}{E}\right)^2}{\frac{1}{2}\left(\frac{iR}{E}\right)^2 + \frac{1}{3}\left(\frac{lR}{E}\right)^3 + \frac{1}{4}\left(\frac{iR}{E}\right)^4 + \cdots}.$$

We then conclude in addition to the above

VI. With given values of R and i the efficiency is a maximum and equals 100 per cent. for $E = \infty$.

VII. With fixed values of E, i and L the efficiency is a maximum and equals 100 per cent. for $R = 0$.

This latter condition is the one present in practice. E is fixed, at least to a certain extent, by the cost and the difficulty of carrying many cells in vehicles or boats. L being also fixed, i must not be less than a certain value or the resultant spark will not be sufficiently intense to ignite the charge. Under these conditions R should be as small as possible with the given inductance.

The above facts can also all be proved by direct differentiation of the formula

$$\eta = \frac{\frac{1}{2}i^2}{I^2 \log \frac{I}{I-i} - iI}.$$

So far, we have neglected entirely the influence of the time of break. It is evident that during this time an additional amount of energy is drawn from the battery. Part of this appears as heat in the spark, and the rest is wasted in I^2R loss in the resistance of the coil.

To calculate the magnitude of this effect, it is necessary to know something about the shape of the current curve during the time that the circuit is being opened. To deduce mathematically the shape of this curve would be difficult if not impossible. It will be seen, however, from the accompanying curves that it is practically a straight line. Assuming this to be the case, the current may be expressed by $i = K(T - t)$ in which T is the time that elapses from the beginning of opening of the circuit until the resistance becomes infinite and t is the time. Since at $t = 0$, $i = i_b$, $K = i_b/T$ or $i = i_b/T(T - t)$.

We may now readily modify our expression for efficiency so as to include this factor. The formula

$$\eta = \frac{\frac{1}{2}\left(\frac{i}{I}\right)^2}{\frac{1}{2}\left(\frac{i}{I}\right)^2 + \frac{1}{3}\left(\frac{i}{I}\right)^3 + \frac{1}{4}\left(\frac{i}{I}\right)^4 + \cdots}$$

may be written

$$\eta = \frac{\frac{1}{2}Li^2}{\frac{1}{2}Li^2 + LI^2\left[\frac{1}{3}\left(\frac{i}{I}\right) + \frac{1}{4}\left(\frac{i}{I}\right)^4 + \cdots\right]}.$$

Adding $\frac{1}{2}Ei_bT$ to both members, subtracting $\frac{1}{3}Ri_b^2T$ from the numerator and dropping the subscript in i_b we get

$$\eta = \frac{\frac{1}{2}Li^2 + \frac{1}{2}EiT - \frac{1}{3}Ri^2T}{\frac{1}{2}Li^2 + \frac{1}{2}EiT + LI^2\left[\frac{1}{3}\left(\frac{i}{I}\right)^3 + \frac{1}{4}\left(\frac{i}{I}\right)^4 + \cdots\right]}.$$

EFFECT OF TIME OF BREAK.

To find the influence of the time of break upon the efficiency we differentiate with respect to T and place the result equal to zero. For this purpose we may regard everything except T and η as constant and write

$$\eta = \frac{K_1 + K_2 T}{K_3 + K_4 T}.$$

Then

$$\frac{d\eta}{dT} = \frac{K_2 K_3 - K_1 K_4}{(K_3 + K_4 T)^2}.$$

This is satisfied for $T = \infty$ and in this case we have

$$\eta = \frac{K_2}{K_4} = \frac{\frac{1}{2}Ei - \frac{1}{3}Ri^2}{\frac{1}{2}Ei} = 1 - \tfrac{2}{3}\alpha.$$

The efficiency with $T = 0$ was

$$\eta = \frac{\frac{1}{2}\alpha^2}{\frac{1}{2}\alpha^2 + \frac{1}{3}\alpha^3 + \cdots} = 1 - \tfrac{2}{3}\alpha - \tfrac{1}{18}\alpha^2 - \tfrac{2}{135}\alpha^3 - \tfrac{47}{620}\alpha^4 \cdots.$$

The first two terms of these expressions are the same, and they differ only by small multiples of $\alpha^2, \alpha^3, \cdots$, etc. Hence, when the ratio $\alpha = i/I$ is small, the efficiency is very little affected by the speed of break. In any case however it becomes somewhat greater as the time of break becomes larger.

THE SPARK COIL WITH IRON CORE.

The coil without iron, while it may have high efficiency as has been shown, is probably impracticable for ordinary purposes. Almost the only use to which primary coils are put, is the ignition of gas engines, for gas lighters, and similar uses. Experience shows that for such work an output of about 60×10^{-4} joules is required. If i be taken as 1 ampere, which is about as much as we would care to use from the ordinary primary battery, we have, assuming 100 per cent. efficiency, $\frac{1}{2}Li^2 = 60 \times 10^{-4}$ or $L = 1.0120/1 = .012$ henry. A coil without an iron core, to have this inductance, would necessarily be of very large size. This is true since it would have to be wound with coarse wire to keep the ratio i/I small when used with a moderate voltage.

The introduction of an iron core gives us three additional losses

to compute. These are the hysteresis loss, eddy current losses at make and at break. We shall designate these respectively by H, E_m and E_b. The expression for efficiency then becomes

$$\eta = \frac{\frac{1}{2}Li^2 - E_b + I_b}{\frac{1}{2}Li^2 + E_m + O_m + H + O_b + I_b},$$

we have now to find expressions for H, E_m and E_b

The hysteresis loss is best found by direct experiment. In the coil here considered three methods have been employed. One is to use a wattmeter and alternating currents, the other consists in obtaining the hysteresis curve or loop for the coil in question and from its area, as from an indicator card, obtaining the actual loss in one cycle. A third method is to compute the loss from the average values obtained from similar samples of iron and used by dynamo designers and others to predetermine hysteresis loss. The three methods give very concordant results. It is well known that this loss for each cycle varies as the 1.6 power of the flux density. The flux density for small values is proportional to the current, consequently we may write $H = \text{constant} \times i^{1.6}$. In the coil considered the constant is equal to 4.69×10^{-4} or $H = 4.69 \times 10^{-4} i^{1.6}$.

The loss due to eddy currents at the make has next to be determined. As the magnetic flux surges back and forth through the wire, currents, first in one direction, then in the other, will be generated and will flow in circles, through the wire. This current wastes power in heat, and, though the amount is small it is not negligible.

Consider a ring of radius r and thickness dr. If φ is the flux through this ring, the E.M.F. induced in it is $e = d\varphi/dt$. But we may write $\varphi = Kai$, where K is a constant depending upon the coil used and a is the area of the ring. Then

$$e = Ka\frac{di}{dt} = \pi r^2 K\frac{dt}{dt}.$$

But

$$i = I(1 - \varepsilon^{-Rt/L}),$$

and

$$\frac{di}{dt} = I\frac{R}{L}\varepsilon^{-\frac{Rt}{L}}.$$

Substituting and dividing by 10^{-8} to reduce to volts, we get

$$e = 10^{-8} K\pi r^2 I \frac{R}{L} \varepsilon^{-\frac{Rt}{L}}.$$

The resistance of the cylinder of which this ring is an element and of a length equal to the length of the coil, say l is equal to

$$\text{Constant} \times \frac{\text{length}}{\text{cross-section}} = \frac{K_1 2\pi r}{edr}.$$

The power loss at any instant is

$$\frac{e^2 dr}{r} = \frac{K^2\pi r^3 I^2 R^2 \varepsilon^{-2Rt/L\cdot l}}{10^{16} \times 2K_1 L^2} dr.$$

There are only two variables in the equation, t and r, and to find the rate of loss of energy in one wire of the core we must integrate with respect to r from 0 to $D/2$, where D is the diameter of the wire. Doing this, we get

$$w = \frac{K^2\pi I^2 R^2 \varepsilon^{-2Rt/L\cdot l}}{10^{16} \times 2K_1 L^2} \int_0^{D/2} r^3 dr = \frac{K^2\pi I^2 R^2 l}{10^{16} \times 128 L^2 K_1} \varepsilon^{-\frac{2Rt}{L}} D^4.$$

To get the loss of energy in one cycle we must multiply this by dt and integrate again from 0 to T_m, where T_m is the time elapsing from the make to the break. This gives us

$$j = \frac{K^2\pi I^2 R^2 l D^4}{10^{16} \times 128 K_1 L^2} \int_0^{T_m} \varepsilon^{-\frac{2Rt}{L}} dt = \frac{K^2\pi I^2 R l D^4}{10^{16} \times 256 L K_1} (\varepsilon^{-\frac{2RT_m}{L}} - 1).$$

which may be readily simplified to

$$\frac{1.36 K^2 R l D^4}{10^{13} L} i(2I - i).$$

We can easily show that

$$K = \frac{10^8 L}{NA},$$

where N is the number of turns of wire in the coil and A is the net area in square centimeters of the core. Then we have finally, for the whole coil, w being the number of wires in the core, loss per cycle due to eddy currents during make in joules equals

$$E_m = \frac{1{,}360 L R l D^4 W}{N^2 A^2} i(2I - i).$$

This is true for any coil, and if the value of L, R, l, D, W, N and A in any particular case be inserted it reduces to the form constant $\times\ i(2I - i)$. In the case of the coil here considered, it becomes $7 \times 10^6\ i(2I - i)$.

To find the losses at the break we must know something about the way in which the current falls off during this time. Assuming as before that the current during break follows a straight line law we may take for our expression for the current $i = -K_3 t$, taking a new origin for the time, for the sake of simplicity.

To get the eddy current loss at break we proceed as before, merely using our new value for i. Thus

$$e = \frac{d\theta}{dt}; \quad \theta = -Kai = -KaK_3 t;$$

then $e = -KK_3 a = -\pi r^2 KK_3$. Taking r as before,

$$dw = \frac{e^2}{r} = \frac{\pi r^3 K^2 K_3^2 l}{2K_1}\, dr.$$

Then rate of loss of energy per wire per cycle equals

$$W = \frac{\pi K^2 K_3^2 l}{2K_1} \int_0^{D/2} r^3 dr = \frac{\pi K^2 K_3^2 l D_1^4}{128 K_1},$$

and the energy loss in joules per cycle for the whole core of W wires, if T_b be the time of break, is

$$E_b = \int_0^{T_b} W dt = \frac{\pi K^2 K_3^2 l T_b D_1^4 W}{128 K_1}.$$

If i is the value of the current at the instant of break, then

$$i = -K_3 T_b \quad \text{or} \quad K_3 = -\frac{i}{T_b} \quad \text{and} \quad K = \frac{L}{NA}.$$

Substituting also for K_1 its value for iron of 9×10^{-6} we get

$$E_b = \frac{\pi L^2 i^2 l T_b D^4 W}{128 N^2 A^2 T_b^2 \times 9 \times 10^{-6}} = \frac{\pi L^2 l D^4 10^3}{1.152 N^2 A^2} \times \frac{i^2}{T_b} \times W.$$

In the coil considered this reduces to

$$7.85 \times 10^{-7} \frac{i^2}{T_b}.$$

The loss of energy in the coil during the break is next to be obtained. As before, $i = -K_3 t$. The loss of energy equals

$$\int_0^{T_b} i^2 R dt = K_3^2 R \int_0^{T_b} t^2 dt = \tfrac{1}{3} K_3^2 R T_b^3.$$

If, as before, $i =$ current at time of break, then $i = -K_3 T_b$ and loss in joules per cycle $= O_b = \tfrac{1}{3} R i^2 T_b$. In the coil here considered $R = 0.725$ ohm, and we get $O_b = 0.242 i^2 T_b$.

We have to obtain now only the added energy furnished by the battery during the break. Since we have assumed that the current varies uniformly from a value i to zero during this time, the average power is evidently $\tfrac{1}{2} Ei$. Then the energy added equals

$$I_b = \tfrac{1}{2} E i T_b.$$

This gives us all the factors needed for the expression for efficiency. As before

$$\eta = \frac{\tfrac{1}{2} L i^2 + I_b - E_b - O_b}{\tfrac{1}{2} L i^2 + I_b + H + E_m + O_m}.$$

Substituting the values we have just found for these factors, we obtain a general expression for the efficiency of any primary coil. Substituting the values of L, R, l, D, N, A, K_4 and W for the coil under consideration the formula will be greatly simplified. In the case of the coil here considered it becomes

$$\eta = \frac{0.02 i^2 + \tfrac{1}{2} E i T_b - 0.242 i^2 T_b - 7.85 \times 10^{-7} \dfrac{i^2}{T_b}}{0.02 i^2 + \tfrac{1}{2} E i T_b + 4.69 \times 10^{-4} i^{1.6} + 7 \times 10^{-6} i (2I - i) + 0.04 I^2 \left[\frac{1}{3} \left(\frac{i}{I} \right)^3 + \frac{1}{4} \left(\frac{i}{I} \right)^4 + \cdots \right]}.$$

To apply this we must know the time of make and break. The former is most readily found by turning the engine over slowly by hand and observing the angle within which contact is made. Knowing the number of revolutions per minute we can easily compute the time of contact. The time of break cannot be obtained in any simple way. It will usually be about one one-thousandth of a second, however, and any small error in estimating it will have but very little influence upon the result. In the case of one engine it

was found that the time of contact was one one-hundredth of a second. Knowing this and the E.M.F. of the battery used, which was in this instance 2.52 volts, we can calculate the value of i.

Substituting in $i = I(1 - \varepsilon^{-Rt/L})$ we get

$$i = \frac{2.52}{0.725}\left(1 - \varepsilon^{\frac{-0.725 \times \frac{1}{100}}{.04}}\right) = 0.584 \text{ ampere}$$

and

$$I = \frac{E}{R} = \frac{2.52}{0.725} = 3.5 \text{ amperes.}$$

Substituting these values in the formula we find the efficiency equal to 82.5 per cent. That is, of the actual energy put into the coil 82.5 per cent. appears in the heat of the spark, the other 17.5 per cent. being wasted in the various losses.

It is of some interest to compare these losses with one another and with the input. They are given in the following table:

Magnetic energy,	$\frac{1}{2}Li^2 = 68.4 \times 10^{-4}$ joules
Energy added during break,	$Ib = 7.35 \times 10^{-4}$ joules
Ohmic loss at make,	$Om = 8.80 \times 10^{-4}$ joules
Ohmic loss at break,	$Ob = 0.83 \times 10^{-4}$ joules
Eddy current loss at make,	$Em = 0.25 \times 10^{-4}$ joules
Eddy current loss at break,	$Eb = 2.68 \times 10^{-4}$ joules
Hysteresis loss,	$H = 2.03 \times 10^{-4}$ joules

Of these the largest is the ohmic loss at the make. This can be reduced by using a battery of higher E.M.F. and a correspondingly short make. This is readily seen to be true, since this loss is dependent upon the ratio of i/I, and increasing the E.M.F. increases I in the same proportion. The ohmic loss at the break is small in any case, and can be still further reduced by shortening the time of break.

In the eddy current losses we have an opposite set of conditions, since each is increased by a decrease of the time of make or break. Hence the conditions for best efficiency here are the exact opposite to those applicable to the ohmic losses. Fortunately, the eddy current loss can be reduced to any desired extent by using finer wires in the core. For a given cross-section of core the loss is in proportion to the square of the diameter. It will be noticed that the loss is, of course, much larger at the break than at the make. Hence it has an especially bad effect, as it takes away materially

from the energy of the spark. Indeed, a short calculation shows that if the break takes place in about thirty-five millionths of a second all the energy would be wasted in the eddy currents.

The magnitude of the hysteresis loss is independent of the time of the make or break, and depends only on the value of the current used and the quality of the iron. To reduce it as far as possible, iron having the smallest hysteresis loss possible should be used.

In any piece of apparatus the efficiency curve is of importance; that is, it is important to see whether the efficiency keeps up well through a wide range of load. I have computed the efficiency for various values of the current i, that is, the current at which the break takes place, the results being given in Table I. The values of i correspond to an intensity of spark from 0 to 0.04 joule. The actual value of i used, it will be remembered, is 0.584 ampere, corresponding to 0.0075 joule per spark. At $i = I = 2.5$ amperes the efficiency would drop to zero.

TABLE I.

Efficiency for Different Values of i.

i	Efficency.
0	77 per cent.
0.1	83 "
0.2	85 "
0.4	83 "
1.0	81 "

The fact that the efficiency stays above 80 per cent. up to about five times the spark intensity used shows that the coil is capable of delivering a spark five times as strong as actually used and still preserve a good efficiency, or, conversely, a very much smaller coil would be capable of doing the work required. At the same time a slightly greater efficiency could be obtained by an even larger coil. However, the gain would be slight and it would seem better to reduce considerably the size of the coils usually used.

To summarize — for the highest efficiency the battery should have a rather high E.M.F. and the contact should be short, so that the current rises only to a small fraction of the value it would attain if given unlimited time. The break should be reasonably short, but not too brief, especially with a coil having wire of a large diameter in the core, and the core should be made of wire having the

smallest possible hysteretic coefficient. For the sake of first cost, the coil could be much smaller than those usually used.

Experimental Work with the Primary Induction Coil.

The preceding theory is, it is believed entirely new, and it seemed desirable to confirm it by direct experimental proof. The method adopted is equally applicable to the ordinary induction coil with a secondary winding, and was used in obtaining various results in connection with such coils.

The first point taken up was an experimental determination of the efficiency of a primary coil. The method adopted was to obtain by means of an instantaneous contact maker, a series of points on the curves of current and E.M.F. of the coil. The summation of the products of E.M.F. and current will give us the energy input and output and consequently the efficiency, and in addition we learn the way the current rises, how it falls off, the inductive rise of pressure at the break, both across the gap and over the coil, the potential difference of the battery, etc. These are gotten directly and having them we may easily compute the values for the curves of power during make and break, the resistance or conductivity of the gap during break, and the internal resistance of the battery.

Some difficulty in obtaining points especially during the break was anticipated. Looking at the sparks, they appear to differ considerably in length, quality, etc., and hence it was feared that it would be impossible to secure consistent results. Some trouble was encountered from this source, but by keeping the contacts clean, this was reduced to a minimum.

The use of the oscillograph to obtain these curves has been suggested, and for merely qualitative results it could undoubtedly be employed to very great advantage. For accurate work, however, it could hardly be used with spark coils. For one thing it would take a relatively large amount of energy from the circuit, especially during the break. For example, the oscillograph manufactured by the General Electric Company requires 0.3 ampere for its operation. At the break the voltage rises to from 50 to 100 volts, and consequently from 15 to 30 watts would be expended in the instrument. Since the maximum wattage of the spark rarely exceeds twice these

figures, it is evident that this method is out of the question for the determination of the efficiency. This objection does not apply to the oscillograph when used on circuits carrying considerable amounts of power.

The arrangement of the apparatus used is shown in Fig. 2. The main circuit is shown in the upper part of the figure and comprises from left to right a battery B a switch S, a rotating break Br, a

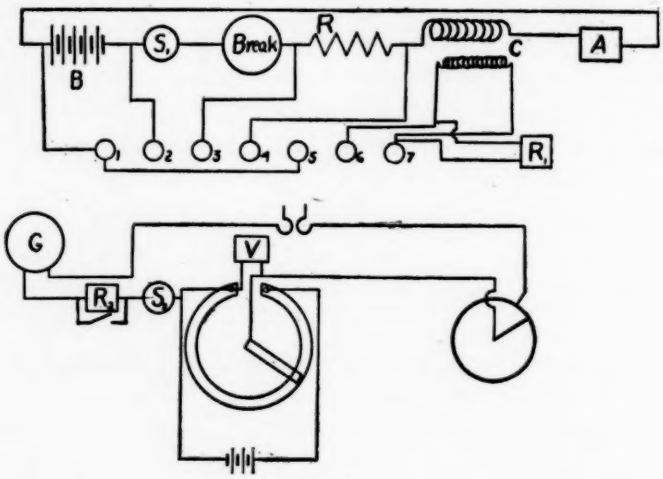

Fig. 2.

non-inductive resistance R of low value, the coil under test C, here shown with a secondary, and an ammeter A. This is the usual circuit of a primary coil as used in gas engine ignition, the only difference being the use of a rotating break and the insertion of the extra resistance R. The former seemed preferable from the standpoint of uniformity of action, and the latter is necessary to obtain the curve of current. It has a resistance, however, of only 0.133 ohm, and in fact amounts to no more than the resistance of the usual wiring of a coil. The fact that we are here operating under actual working conditions is very important.

From points on the circuit as shown, connections are made to a row of mercury cups. By means of the fork-shaped arrangement shown just below the mercury cups, connection can be made to any part of the circuit. Thus taking them in order from left to right, we can connect with the battery, the break, the non-inductive

resistance, the primary of the coil and the secondary of the same. With the primary is included the ammeter, which may be considered an extra inductance and resistance added to the coil. Its influence, however, is negligible.

The contact maker marked *C.M.* is of the usual form used in investigating E.M.F. waves. It is rigidly connected to the shaft of the rotating break and the whole is operated by a 1 H.P. motor. The current for the motor is taken from a storage battery. Thus the pressure and hence the rate of rotation are very uniform. Changes of speed are provided for by varying the number of cells connected to the armature. The field is kept constant. In this arrangement no resistance is used, hence the rotation at low speeds is much more uniform than would otherwise be possible.

S.R. (Fig. 2) is a shunt rheostat. This is used to give any desired potential difference from zero to that of the battery employed. In the actual setup, a slightly more complicated arrangement was used. It was necessary to have a variable voltage ranging from zero to about one hundred volts, and instruments capable of reading accurately any voltage in this range. This was accomplished by using two voltmeters, each having two scales. One was a milli-voltmeter giving a full scale deflection with 0.03 or 0.3 volt depending upon which scale was used. The other voltmeter had ranges of 3 and of 150 volts. Any one of these four scales could be readily connected as desired. For convenience in getting low voltages, two batteries instead of one as shown were used and either could be readily thrown in as desired. For a fine adjustment, a low adjustable resistance was used in series with the battery. This is also omitted in the figure. The instantaneous voltage across any part of the circuit can be balanced against the voltage at the terminals of the shunt rheostat, and the curves thus determined point by point. Current curves are obtained by measuring the voltages over the non-inductive resistance and dividing by the resistance.

This method while rather laborious, has four very important advantages over any other. The first is that the circuit is working under absolutely normal conditions, the resistance of R and that of the ammeter being no more than would usually be present in

practice. Secondly the voltages are read directly and on ordinary voltmeters. Therefore the accuracy is great, no computation is required and the chance of error is slight.

The third and greatest advantage is that absolutely no current is taken from the circuit in making the measurements. This follows of course from the fact that this is a balance method, the energy to operate the voltmeter being taken from the auxiliary battery. This is very important as the amount of energy carried is small and if any were required to operate the instruments, the error would be considerable.

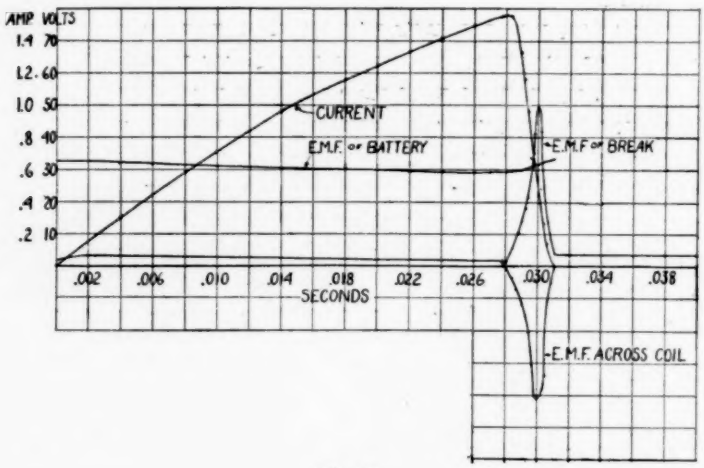

Fig. 3.

It should also be pointed out that changes in the force with which the brush of the contact maker presses on the rotating steel piece, do not affect the accuracy of the method. A poor contact has simply the effect of introducing a little extra resistance in the galvanometer circuit, but does not at all change the balance.

Let us now consider some of the results obtained. Fig. 3 shows the complete curves of a coil during one cycle and Fig. 4 shows the portion of the curves during the break on an enlarged scale. As will be seen the current curve starts from zero, rises gradually to its maximum value, and descends rapidly to zero during the break. The duration of make was 0.029 second and that of the break, 0.0030 second. During the make the current, as was pre-

viously pointed out, follows the law $i = I(1 - \varepsilon^{-Rt/L})$. The values of the constants were determined for this case and were as follows, $E = 3.33$, R (of entire circuit) $= 1.276$ and $L = 0.0422$. The curve that would be obtained using these values, differs inappreciably from the one in Fig. 3.

Near the top of the curve it bends over gradually. This is due to the fact that the brush is beginning to slide off from the contact, thus increasing the resistance. As soon as the break actually occurs, the current drops very rapidly and the curve is practically a straight line. This is seen better from Fig. 4. The fact that

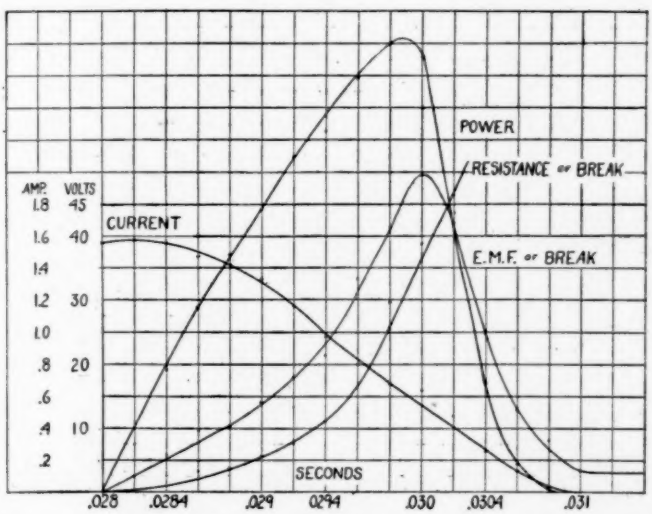

Fig. 4.

the current during the break follows practically a straight line is of importance, as this assumption was made in the theoretical part of this investigation.

The curve of E.M.F. across the break is next in importance. From the moment of contact until the break begins to take place the potential difference is very small, amounting only to the product of the current and the resistance of the contact. It was accurately measured and found to increase according to the same law as the current, as was of course to be expected. The values, however, are too small to show in the curve. At the instant of the break, the stored magnetic energy of the coil is liberated and shows itself in

the sudden and large rise in voltage. In this case the maximum voltage was 49.5 volts. After reaching its maximum it very rapidly drops to 3.33 volts, the E.M.F. of the battery, which is of course applied across the gap as long as the the circuit is open.

The curve of the E.M.F. over the coil is next to be considered. At the instant of make it jumps to approximately the E.M.F. of the battery, and then gradually decreases during the make. At the instant of break, the induced E.M.F. is suddenly reversed by the decrease of the lines of induction and a peak much like that of the *Eb* curve is produced, but in the opposite direction.

The last curve is that of the potential difference of the battery. It of course drops off gradually during the make as the current is increased and again rises during the break to its former value. Since the sum of all the E.M.F.'s in any closed circuit is zero, calling the E.M.F. over the non-inductive resistance *Er*, we should have $Eb = Er + Ebr + Ec$ each being of course taken with its proper sign. This is a good test of the correctness of the work and as is seen from the curves is very accurately fulfilled.

In Fig. 4 two additional curves are plotted those of power and resistance of break. These are computed curves, the latter being obtained by dividing *Eb* by *i*. It of course starts from almost zero, and quickly rises, becoming infinite at the instant of complete break.

The other curve *W* is the curve of power. It is obtained by multiplying together the instantaneous values of current and E.M.F. during the break. The maximum of this curve is 31.4 watts. This is very important as it is probable that good ignition is dependent upon the maximum of this curve rather than upon its total area.

We are now in a position to solve our main problem, the determination of the efficiency. This may be obtained either for the whole circuit including the battery or for the coil alone. The former is slightly simpler and will be here considered.

The rate of input of energy is evidently *Ei* where *E* is the E.M.F. of the battery and *i* is the current. This lasts during both make and break. Since *E* is constant, the total input is simply (average *i*) $\times E \times t$, where *t* is time of both make and break. Here average $i = 0.895$ ampere, $E = 3.33$ volts and $t = 0.0318$ second. Hence $W = 0.895 \times 3.33 \times 0.0318 = 0.0942$ joule.

Similarly the output is given by (average ei) $\times$ tb, in which e is the E.M.F. across the break only. Average ei is the same as average W, or the average of the curve W. This from the curve is found to be 16.41 watts, and $tb = 0.00309$ second. Hence the output is 0.0508 joule. Finally efficiency equals

$$\frac{\text{output}}{\text{input}} = \frac{.0508}{.0942} = 54.0 \text{ per cent.}$$

A comparison of this value with that obtained according to the theory previously developed is of interest. Calculating the various quantities by the method there explained we obtain the following where all quantities are expressed in joules :

Magnetic energy,	0.0514
Energy added during break,	0.0080
Eddy current loss during make,	0.00004
Eddy current loss during break,	0.0006
Ohmic current loss during make,	0.0385
Ohmic current loss during break,	0.0032
Hysteresis loss,	0.0010

Substituting in the formula for efficiency gives

$$\eta = \frac{514 + 80 - 32 - 6}{514 + 80 + 10 + 0.4 + 385} = \frac{556}{989} = 56.2 \text{ per cent.}$$

This computed value agrees very well with the observed value of 54 per cent.

It should be pointed out that this coil was not operating under the best conditions. The energy in the spark, 0.0508 joule, was about three times as much as is needed for good ignition. Hence it would have been practicable to cut down very greatly the time of make. This would have reduced the ohmic loss of the coil, which as may be seen above is much greater than any of the others. With a contact of about $\frac{1}{100}$ second the efficiency of the coil as was shown before is about 85 per cent.

Fig. 5 shows curves for another coil. This coil was much larger than the first one, contained more iron, and consequently had an inductance about twice as great. The resistance, however, was only slightly greater. The speed of the rotating break was increased so that the make was about two thirds as long as before. The influence of the inductance is clearly seen in the much slower rise of the

current. The voltage at break on account of the high inductance and the quick break rises higher than before reaching 67 volts.

At the right are given on an enlarged scale the curves of volts, watts, amperes and ohms at break. These show the same general shapes as before. The input of the coil is of course less and the efficiency works out to be 43.2 per cent.

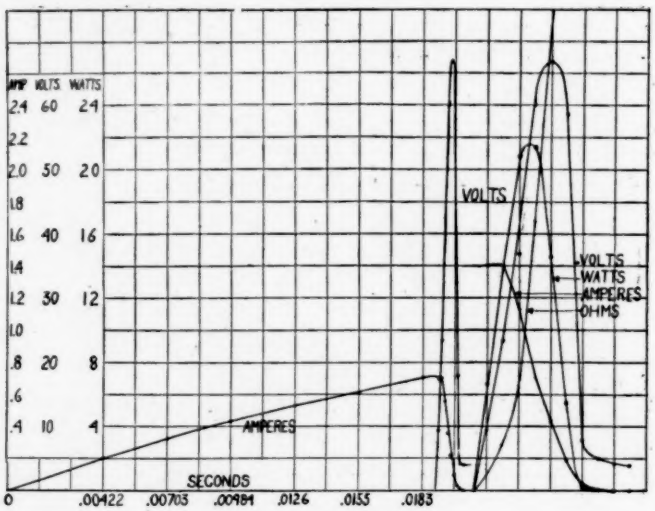

Fig. 5.

In Fig. 6 we have on the other hand the curves of a coil having an inductance of only 0.0121 henry; about one fourth of that of the coil of Fig. 3 and about one seventh of that of Fig. 5. This was operated with a voltage of only 2.072 that of one storage cell; in fact the coil was wound to test the possibility of operating a primary spark coil with a very low voltage. The time of make and break are about the same as in Fig. 4. The current rises very rapidly on account of the low inductance. The voltage at break, however, is only about half as great as in the other two cases, but the current is so much larger that the maximum watts and hence presumably the igniting power is about the same. The efficiency is slightly higher, being 49.5 per cent.

THE INDUCTION COIL OR SECONDARY COIL.

In studying the induction coil we shall first consider the simple case of a coil with the secondary open and no condenser used in

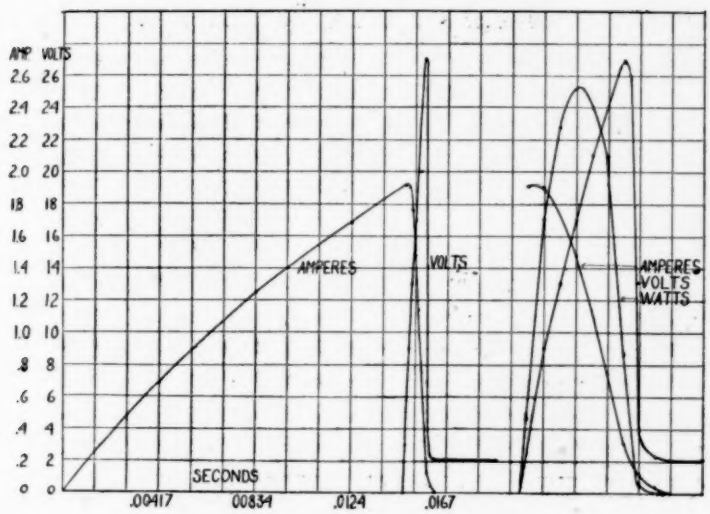

Fig. 6.

the primary. Under these conditions, the electrical actions are in all respects similar to those of the primary coil. To be sure the secondary winding has a certain capacity, and the E.M.F. induced in it causes a slight displacement of electricity. This effect is greatest in large coils wound with very fine wire, but even there it is entirely negligible. With coils for very high frequency, say in the neighborhood of 100,000 cycles per second, the case is entirely different, and this capacity may have a very decided effect upon the voltage produced. For the present we shall consider only coils in which the frequency does not exceed 1,000 cycles per second, and this effect will consequently be neglected.

For the coil then without condenser and whose secondary is open no farther investigation is necessary. Figs. 4 and 5 show all the actions taking place with the single exception of the curve of secondary E.M.F. This is, however, readily derived. Since we have assumed that there is no current in the secondary, it will obviously have no effect upon the primary, and the primary current during make will follow the usual law $i_p = I(1 - \varepsilon^{-Rt/L})$. We then have

$$es = M\frac{di_p}{dt} = \frac{MRI}{L}\varepsilon^{-\frac{Rt}{L}} = \frac{M}{L}E\varepsilon^{-\frac{Rt}{L}}.$$

This gives us the value of the secondary E.M.F. at any instant dur-

ing the make. It starts with the value ME/L at $t = 0$ and gradually decreases, becoming practically zero for large values of t.

Most commercial induction coils are provided with condensers connected across the break. These serve in general two purposes. First, they prevent largely the arcing which would otherwise occur at the break and secondly they increase in general the E.M.F. induced in the secondary. In addition they may or may not set up oscillations in the circuits. We shall now examine these functions somewhat in detail.

During the make the condenser is short circuited and hence it has obviously no effect. We may then neglect this part of the cycle and concern ourselves with the break only. In general the current during the break becomes oscillatory, and dies out gradually. If the break be instantaneous we have the following equations in which all the quantities refer to the primary.

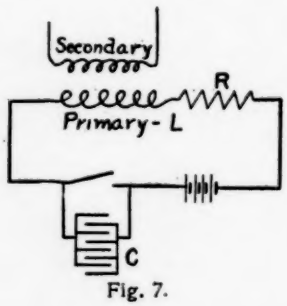

Fig. 7.

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E$$

where q is the quantity of electricity in the condenser, E is the E.M.F. of the battery and C is the capacity of the condenser. dq/dt is the current i and the equation is the same as before except for the term q/C. This is the potential difference over C the condenser. The arrangement of the circuit is shown in Fig. 7.

The solution of the above well-known differential equation is

$$i = \frac{dq}{dt} = Qe^{at}[\,\omega \cos \omega t + a \sin \omega t\,]$$

where

$$a = -\frac{R}{2L}, \quad \omega = \sqrt{\frac{1}{CL} - \frac{R^2}{4L^2}},$$

and the time of one oscillation is given by

$$T = \frac{1}{n} = \frac{2\pi}{\sqrt{\frac{1}{CL} - \frac{R^2}{4L^2}}}.$$

Usually in practice $R^2/4L^2$ is very small compared with $1/CL$ and may be neglected. In this case we have $T = 2\pi\sqrt{LC}$. In practice ω is usually of about the order 10^3 while α is purposely kept low. Hence we may usually neglect the last term and we get $i = Q\omega\varepsilon^{\alpha t} \cos \omega t$. It is evident that just at the instant of break $i = i_b$ and since $t = 0$ we have $i = Q\omega$ or substituting,

$$i = i_b \varepsilon^{-\frac{Rt}{2L}} \cos \frac{t}{\sqrt{LC}}.$$

This means that the current oscillates with a gradually decreasing amplitude. In fact it dies out gradually in just the same way that it grows to its final value during make, but only half as fast.

The primary E.M.F. is given by $L \cdot d^2q/dt^2$ as shown above and the secondary E.M.F. by $M \cdot dq^2/dt^2$. If as before α is negligible compared to ω we have $e_p = LQ\omega^2\varepsilon^{\alpha t} \sin \omega t$ and $e_s = -MQ\omega^2\varepsilon^{\alpha t} \sin \omega t$. That is in case the decrement is small the E.M.F. and the current differ 90° in phase.

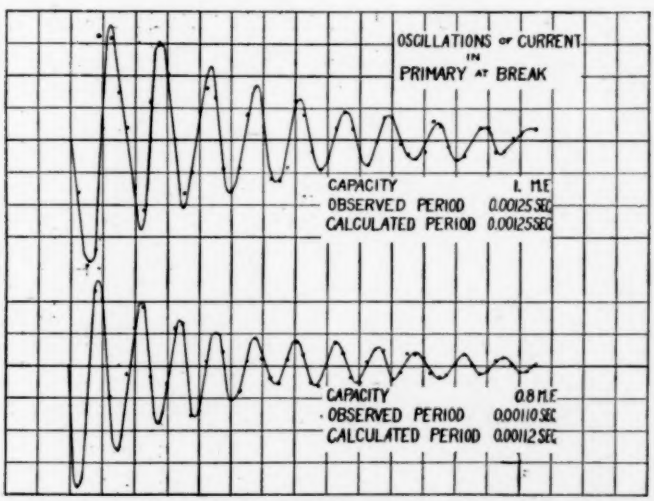
OSCILLATIONS OF CURRENT IN PRIMARY AT BREAK
CAPACITY 1. M.F.
OBSERVED PERIOD 0.00125 SEC.
CALCULATED PERIOD 0.00125 SEC.
CAPACITY 0.8 M.F.
OBSERVED PERIOD 0.00110 SEC.
CALCULATED PERIOD 0.00112 SEC.

Fig. 8.

The first point taken up experimentally was a comparison of the calculated time of oscillation with the observed time. Fig. 8 for example shows a series of points on the curve of primary current of a coil having an inductance of 0.0422 henry, used with a condenser

of 1 microfarad capacity. Points could be readily taken for a long distance on the curve as shown. The number that could be obtained was, however, limited by the rotating break closing the circuit again. The points were taken one degree apart and since the contact maker was revolving 393 times per minute this corresponded to $60/393 \times 720 = 0.000213$ second. The average length of a wave in Fig. 9 is 5.88 degrees or 0.00125 second or 8,000 complete vibrations per second. The resistance of the circuit was 1.28 ohms, and substituting in the formula we get as the calculated period 0.00125 second which is the same result as before.

The lower curve of Fig. 8 is a curve of primary current for the same coil but with a smaller capacity. The two curves of Fig. 9

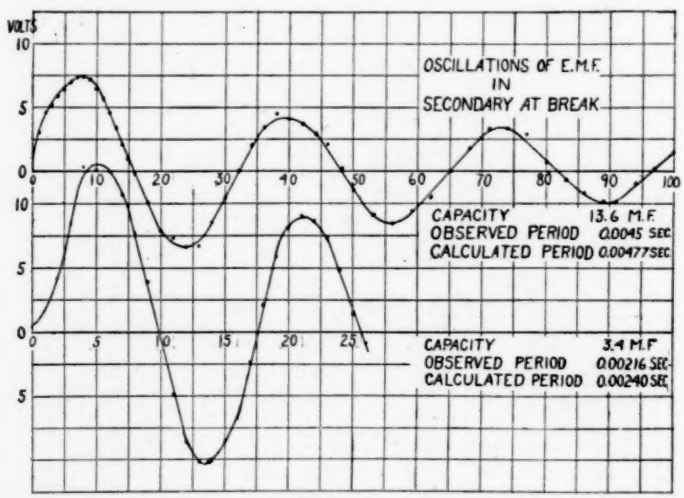

Fig. 9.

are curves of secondary E.M.F. in the same coil. The frequency of oscillation is of course the same in the secondary as in the primary and the calculated and observed times show good agreement. These four curves with the exception of the lower one of Fig. 9 are all plotted to the same scale of time and illustrate well the great variation in frequency caused by the condenser.

In the remaining curves, in order to save time, only a few points near the beginning of the curve were taken, then the contact maker was moved ahead a certain number of degrees, the number of

waves passed over counted, and a few more points taken. This supplied at least two crossings, and the number of waves between the crossings, and consequently all the data needed to determine the time of vibration were at hand. Table II. gives the computed

TABLE II.

Capacity in M.F.	Computed Time.	Observed Time.
1.0	.000125	.000125
.8	.000116	.000110
.6	.000100	.000093
.4	.000082	.000078

and observed times for a number of different values of the capacity. It will be noted that in every case except the first, the observed time is less than the computed. This is probably due to the fact that, especially with small values of the capacity, there is some burning of the contacts at the break. This burning and roughening of the surface causes the break to occur earlier and consequently has the effect of moving the later points farther to the left than they should go, thus causing the apparent discrepancy.

EFFECT OF CAPACITY UPON THE SECONDARY E.M.F.

At the moment of break the energy stored in the coil is $\frac{1}{2}Li_b^2$ and that in the condenser is zero. Assuming that there is no loss at the break, *i. e.*, no sparking, practically all of this is transferred to the condenser a quarter of a period later, from whence it again surges back into the coil and so continues until it is all dissipated in various losses. When the energy is stored in the condenser its value is $\frac{1}{2}E_c^2C$ and since as explained above this is practically equal to $\frac{1}{2}Li_b^2$ we may write

$$\tfrac{1}{2}E^2C = \tfrac{1}{2}Li_b^2$$

or

$$E_c = i_b\sqrt{\frac{L}{C}}.$$

But the maximum E.M.F. over the coil is the same as that over the condenser since they are directly connected together, hence we have for the primary

$$E_b = i_b\sqrt{\frac{L}{C}}.$$

If the magnetic leakage be neglected the primary and secondary E.M.F.'s are in the ratio of their respective turns or $E_s/E_p = N_s/N_p$. Hence we have as the final expression for the secondary E.M.F.

$$E_s = i_b \frac{N_s}{N_p} \sqrt{\frac{L}{C}},$$

and we see that both the primary and secondary E.M.F. vary inversely as the square root of the capacity.

On the other hand we have to consider the loss of energy in sparking at the break. It is of course evident that the E.M.F. across the break and that across the condenser are the same. The larger the condenser the lower the E.M.F. and the less the spark-

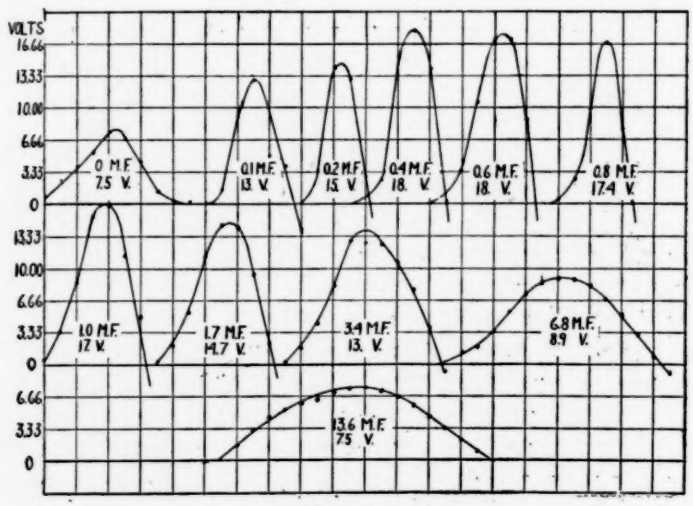

Fig. 10.

ing. It is then quite evident that we have here two conflicting elements and it appears probable at once that a compromise will give the best results. This is already known to be the case[1] and is shown very clearly in Fig. 10.

This figure shows curves of secondary E.M.F. for gradually increasing values of the capacity. With zero capacity the E.M.F. rises to 7.5 volts and gradually falls. In this case there is no oscillation. With 0.1 microfarad the E.M.F. rises to 13 volts and we

[1] Mizuno, Philosophical Mag., 5th Series, No. 45, p. 447.

get a series of oscillations in the secondary. This is also the case in all the following curves, but to save space only the first half wave is shown. This in every case gave the greatest value of the voltage. The succeeding curves show increasing values of the E.M.F. up to a capacity of 0.4 microfarad and beyond this point they decrease regularly. In this particular case then 0.4 microfarad is the best capacity if maximum secondary E.M.F. be the only consideration. If efficiency also is to be considered, a larger value of the capacity would be desirable.

It is possible by means of the formula already derived to calculate the values of the E.M.F. that should be developed in the secondary at break. This is done by means of the formula

$$E_s = \frac{N_s}{N_p} i_b \sqrt{\frac{L}{C}},$$

and it furnishes us with the means of computing the E.M.F. of any induction coil. To do this it is necessary to know the values of i_b, L, N_s, N_p and C. These can, however, be readily measured in any particular coil.

In the case of the coil used in obtaining these curves, the current at break was 1.20 amperes, and the other values are as given before. In the case of the last curve of Fig. 10 we have

$$E_s = \tfrac{588}{504} \times 1.20 \sqrt{\frac{.0422}{.0000136}} = 7.70 \text{ volts.}$$

The observed value as shown by the curve is 7.5 volts, which agrees very well with the calculated value.

TABLE III.

Speed of Breaking = 80 Inches per Second.

Capacity in M.F.	Maximum Observed.	Computed E.M.F.
0.0	7.5	infinity
0.1	13.0	91.2
0.2	15.0	64.4
0.4	18.0	45.5
0.6	18.0	37.2
0.8	17.4	32.2
1.0	17.0	28.1
1.7	14.7	22.1
3.4	13.0	16.5
6.8	8.9	11.0
13.6	7.5	7.8

In the same way the E.M.F.'s for the other values of the capacity were calculated. The results as well as the observed maxima of the waves are given in Table III. and the same quantities are plotted in Fig. 11. They show good agreement for the larger values of the capacity, the observed E.M.F. being always less than the computed one on account of the loss at the break. The dis-

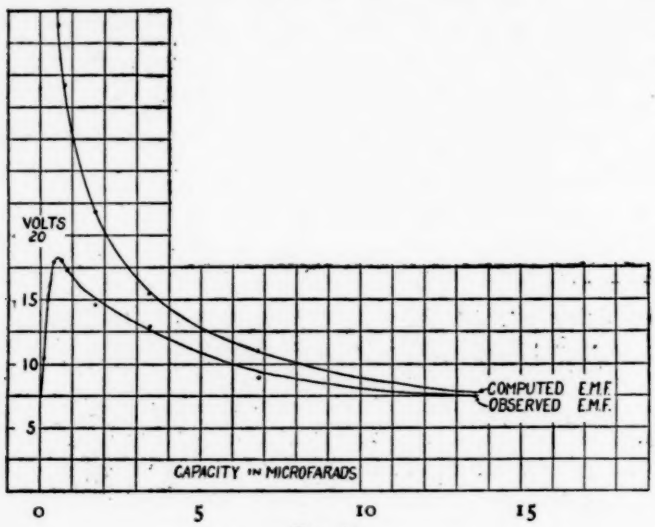

Fig. 11.

crepancy becomes greater as the capacity becomes less, on account of the condenser being less able to suppress the sparking. It is apparent from the curve that if the object be to get the highest possible secondary E.M.F., *i. e.*, the greatest spark length, the condenser should have a value of 0.4 M.F. However with this capacity there is considerable sparking and the efficiency is rather low. Consequently for general purposes a somewhat larger capacity would be preferable.

Effect of Current in the Secondary.

So far in these experiments, no current has been taken from the secondary, that is the coil has been on open circuit. We have now to enquire into the changes that are produced in practice, when the secondary is used to supply current. Figs. 12 and 13 were taken to show the action in this case.

The coil used had a core consisting of about 600, No. 16 soft iron wires, forming a bundle $6\frac{3}{8}''$ long by $1\frac{3}{4}''$ in diameter. The secondary and the primary each contained 252 turns of No. 13 wire. The secondary was wound outside of the primary and thus had a higher resistance. Otherwise they were alike.

This plan was adopted to keep the secondary E.M.F. within the limits of about 100 volts for the sake of ease of measurement.

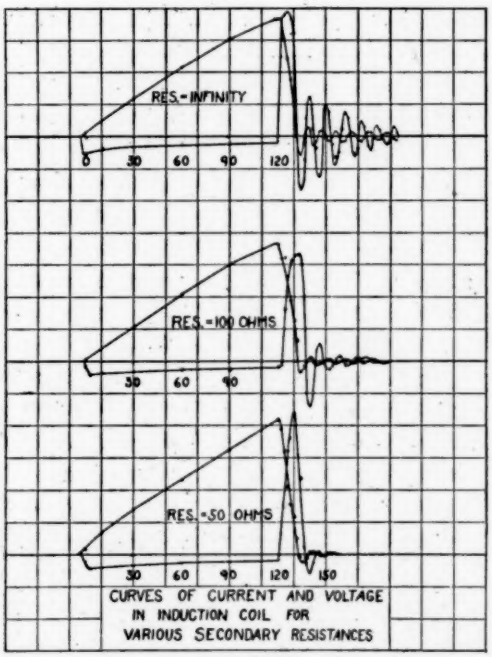

Fig. 12.

The first curve of Fig. 12 shows the variations of secondary E.M.F. and primary current in the coil with open secondary. The primary current as before rises gradually, breaks and dies away in a series of oscillations of decreasing amplitude. The secondary E.M.F. at make jumps up almost instantly to its maximum make value. This value is practically equal to the battery voltage multiplied by the ratio of the secondary to the primary turns. The equation of this curve has been obtained and is

$$e_s = \frac{M}{L} E \varepsilon^{-\frac{Rt}{L}}.$$

If the leakage is small $M = \sqrt{L_p L_s}$ where L_p and L_s are the inductances of the primary and secondary respectively; hence

$$e_s = -E\sqrt{\frac{L_s}{L_p}}\varepsilon^{-\frac{Rt}{L}},$$

and since the inductance is proportional to the square of the number of turns this becomes

$$e_s = E\frac{N_s}{N_p}\varepsilon^{-\frac{Rt}{L}}.$$

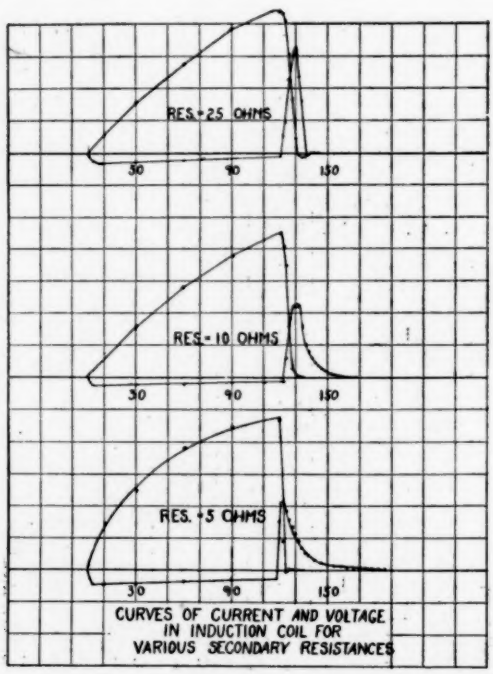

Fig. 13.

At $t = 0$ this becomes $E \cdot N_s/N_p$ and decreases gradually as t increases, in fact approaching zero according to the same law as i approaches I. The actual curve agrees closely with the theoretical.

This make E.M.F. is usually small compared with that at the break. It may be made smaller or larger by decreasing or increasing E the battery voltage. For certain purposes, notably in X-ray work it is important to keep this make E.M.F. low in order to avoid

current in the wrong direction. Here it would be well to keep E as small as possible. A coil used with a Wehnelt interrupter on a 110-volt or 220-volt circuit naturally gives much trouble from inverse current on this account. Such a coil should work much better on an alternating than on a direct current, since the current would in general not rise so rapidly.

At the break the conditions are quite different. We have shown that here we have

$$i = i_b \varepsilon^{-\frac{Rt}{2L}} \cos \frac{t}{\sqrt{LC}} \quad \text{and} \quad e_s = -Mi_b \frac{1}{\sqrt{LC}} \varepsilon^{-\frac{Rt}{2L}} \sin \frac{t}{\sqrt{LC}}.$$

These equations show that both the primary current and the secondary E.M.F. will die out in a series of oscillations, the rate of dying out, *i. e.*, the logarithmic decrement will be half that of the current and E.M.F. at the make since here the coefficient was $\varepsilon^{-Rt/L}$. Moreover the current and E.M.F. will be so related to one another that one will be zero when the other is a maximum, *i. e.*, they differ 90° in phase. All these conclusions are verified from the curves and are all obvious except the second. To show this let us examine the ratio of the maxima of the two successive E.M.F. waves. For this we may take $\omega t = 0$ and $\omega t = 2\pi$ or $t = 0$ and $t = 2\pi/\omega$; we then have

$$\text{Ratio} = \frac{-Mi_b\omega\varepsilon^{-R0/2L} \sin 0}{-Mi_b\omega\varepsilon^{-R\pi/L\omega} \sin 2\pi} = \frac{\varepsilon^{-0}}{\varepsilon^{-R\pi/L\omega}} = \varepsilon^{\frac{R\pi\sqrt{CL}}{L}} = \varepsilon^{R\pi\sqrt{\frac{C}{L}}}.$$

This then should be the ratio of two successive waves and substituting the values of the quantities we have Ratio $= 2.72^{.0391} = 1.04$. Hence each succeeding wave should be about 4 per cent. lower than the preceding one. The actual decrease is about 25 per cent. instead of 4 per cent. The difference is largely due to the loss in eddy currents which act just like a load on the secondary.

As soon as current is taken from the secondary the conditions change. The solution of the equations becomes quite complicated and will not be given here.[1] It is evident however, that if the secondary were connected to a condenser, the current in the secon-

[1] A fairly complete treatment of these equations may be found in Fleming's "The Alternating Current Transformer," pages 183 to 191 or in an article by J. E. Ives, PHYSICAL REVIEW, vol. 14, p. 280, and vol. 15, p. 7.

dary would tend to oscillate in the same way that that in the primary does. The two currents would not necessarily have the same frequency nor the same decrement. Owing, however, to the mutual action of the primary and secondary, both currents would take up an oscillation which would be the sum of two oscillations of different frequencies and decrements. In the case of a secondary connected to a non-inductive resistance, the secondary current of itself will not oscillate.

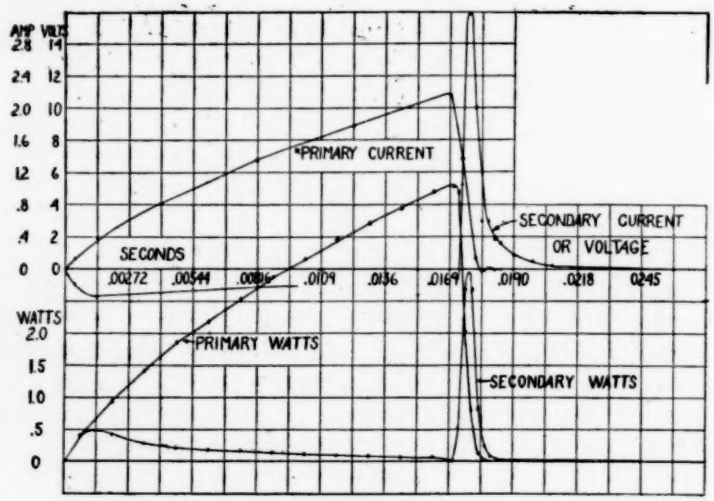

Fig. 14.

Hence, on account of the mutual induction of the two coils it will tend to prevent the primary current from oscillating. This action of the secondary current will obviously be the more powerful the less the secondary resistance, that is, the greater the secondary current.

Figs. 12 and 13 show the effect of increasing gradually the current taken from the secondary. The resistance used was non-inductive, hence the curve of terminal secondary E.M.F. may be equally well considered the curve of secondary current, since the current and E.M.F. are in phase. In all the curves we notice at the moment of make a rounding of the curve of secondary E.M.F. or current. This is of course due to the inductance of the secondary tending to prevent the current from rising too rapidly.

In the second curve of Fig. 12 with 100 ohms secondary resistance oscillations are present although much reduced and in the third curve for which the secondary resistance was 50 ohms this reduction is still more marked. In the curves of Fig. 13 with respectively 25, 10 and 5 ohms the oscillations have disappeared although a slight trace is still present in the primary current. It will be noted that the secondary E.M.F. and current attain their maximum values at about the time the primary current becomes zero. It is evident that the magnetism of the core is not zero, being maintained by the secondary current, and this magnetism now proceeds to dissipate its energy, by producing current in the secondary. In fact there is no other way for it to be dissipated, providing oscillations are not present, when it would be largely wasted in eddy currents and I^2R losses.

In the induction coil employed as it usually is to produce a spark, the curves would be somewhat different. In general no current would flow during the make, and the curve of decrease of current would be somewhat steeper, since the resistance depends upon the current flowing.

We may regard the induction coil from a slightly different standpoint. By considering only the case most frequently met with in practice, that of a non-inductive receiving circuit without capacity, the theory becomes simpler. If we further suppose that the receiving circuit has a low enough resistance so that oscillations are entirely prevented, it becomes possible to present the entire theory of the coil in a very definite and simple form, the only uncertainty being that connected with the action during the break. This period is very short compared with the duration of the break current of the secondary. The theory of the coil under these circumstances has been treated by Fleming in the book before referred to and it is there shown that during the make the currents are represented by

$$i_p = I - \frac{I}{2}\left(\varepsilon^{-\frac{Rt}{L+M}} + \varepsilon^{-\frac{Rt}{L-M}}\right)$$

and

$$i_s = -\frac{I}{2}\left(\varepsilon^{-\frac{Rt}{L+M}} - \varepsilon^{-\frac{Rt}{L-M}}\right).$$

The inductance of the primary and secondary are here considered as equal.

During the break some uncertainty exists. If, however, the break be assumed to follow a straight line law, and this seems to be amply justified by the curves taken, we have the very simple equation,

$$0 = R_s i_s + L\frac{di_s}{dt} + M\frac{di}{dt}.$$

Since we represent i as before by

$$i = i_b - \frac{t}{T} i_b, \quad M\frac{di}{dt} = -\frac{i_b}{T}. \quad M = a \text{ constant.}$$

Hence

$$M\frac{i_b}{T} = R_s i_s + L\frac{di_s}{dt},$$

is the equation of the secondary circuit. This is the ordinary Helmholtz equation and its solution is

$$i_s = \frac{Mi_b}{TR_s}\left(1 - \varepsilon^{-\frac{R_s t}{L_s}}\right).$$

To find the value of i_s at the instant of complete break we have only to substitute $t = T$. We are especially interested in the case when the time of break T is very nearly zero. The expression for i_s becomes of the form $0/0$ for $T = 0$, hence writing

$$i_s = \frac{Mi_b}{R_s} \cdot \frac{(1 - \varepsilon^{-R_s T L_s})}{T}$$

and differentiating both members we get

$$\frac{\frac{R_s}{L_s}\varepsilon^{-\frac{RT}{L_s}}}{1} \cdot \frac{Mi_b}{R_s},$$

which for $T = 0$ is equal to

$$\frac{Mi_b}{R_s} \cdot \frac{R_s}{L_s} = \frac{Mi_b}{L_s}.$$

If as is very nearly the case

$$M = \sqrt{L_s L_p} \quad \text{and} \quad \frac{L_s}{L_p} = \frac{N_s}{N_p^2}$$

we may write

$$i_{sm} = \sqrt{\frac{L_p}{L_s}} \cdot i_b = \frac{N_p}{N_s} i_b.$$

This result might have been obtained more simply by consideration of the energy involved. The work stored in the primary at the instant of break is $\frac{1}{2}L_p i_b^2$. If the break be instantaneous and hence no energy be wasted in the process, all this energy must be transferred to the secondary. We then have at once

$$\tfrac{1}{2}L_p i_b^2 = \tfrac{1}{2}L_s i_s^2 \quad \text{or} \quad i_s = \sqrt{\frac{L_p}{L_s}}\, i_b.$$

This is a rather surprising result at first sight, since the E.M.F. is independent of the capacity of the condenser and of the resistance in the secondary circuit. Moreover, it depends upon the inverse ratio of the turns N_p/N_s. It should be contrasted with the expression for the E.M.F. at break

$$e_{sm} = \frac{N_s}{N_p} i_b \sqrt{\frac{L_p}{C_p}}.$$

This arises from the fact that while the E.M.F. generated is in proportion to N_s the inductance of the secondary at the same time increases in proportion to the square of N_s and hence an increase in N_s actually means a decrease of the maximum momentary current.

It is a fact that many makers of large induction coils for X-ray work are winding the secondaries with much coarser wire than formerly and the users claim that such coils pass much more current in the proper direction and much less inverse than those of the conventional design.

When the break is completed the influence of the primary upon the secondary ceases almost entirely. We have then merely to consider the dying away of a current in an inductive circuit. The equation is $0 = R_s i_s + L_s di_s/dt$. The well-known solution of this is $i_s = i_{sm}\varepsilon^{-R_s t/L_s}$. Theoretically this would require an infinite time for the current to die away. As may be seen by Figs. 13 and 14 the actual time was comparatively long, and traces of current could be detected for some time beyond that shown.

The curves of Figs. 13 and 14 confirm the preceding theory very closely, with the possible exception of that relating to the current during break. The E.M.F. generated during this time is so great and the consequent rise of current is so rapid that it is difficult to follow it. However, as nearly as can be seen from the curves, the shape is apparently that called for by the theory.

EFFICIENCY.

The efficiency of an induction coil may be obtained in the same way as in the case of the primary coil. An examination of the curves of Figs. 11 and 12 indicates that the maximum secondary output will be obtained with some value of the secondary resistance between 5 and 10 ohms. It was estimated that 8 ohms would probably be the resistance giving the greatest output and the curves of Fig. 13 were taken with this value. The upper curves, those of primary current and secondary voltage, were taken directly. The curves of watts in the primary and secondary were computed from the readings, the points of the secondary watts being given by E_s^2/R_s. This latter would not be possible had R_s not been non-inductive. It should be noted that the secondary watts at the break are plotted on a scale only one tenth as great as is used in the other curves. It is interesting to note that just after the make the curve of secondary watts rises almost as fast as that of primary watts, or the output is almost as great as the input. Hence the efficiency is high and the relative amount of energy delivered during the make is great. If it is desirable to get as much energy as possible from the coil during the make, the contact should be short and the battery voltage high. The usual requirement is just the reverse of this, as was pointed out, but for medical coils, such a distribution of energy between the make and break is frequently preferable.

The efficiency is readily obtained from the curve by dividing the area under the curve of secondary watts by that under that of primary watts. This gives here the following results:

Input during make,	0.0417 joule
Input during break,	0.0032 joule
Output during make,	0.0031 joule
Output during break,	0.0193 joule
Total input,	0.0449 joule
Total output,	0.0224 joule

The efficiency is then .0224/.0449 or 50 per cent.

If the coil were used to produce a spark and the gap resistance were high enough so that no current passed during the make, the output would be approximately 0.0193 joule. The input would

not be greatly affected and the resulting efficiency would be 0.0193/0.0449 or 43 per cent.

In any given coil it is possible to predetermine, to a certain extent the output, at least we can set an upper limit to the energy of each spark, and thus compare the igniting power of various coils. This follows from the fact that the energy in the spark cannot possibly exceed the stored magnetic energy of the coil. That is, it cannot exceed $\frac{1}{2}Li_b^2$. L and i_b are determined in the manner previously explained. For example in the present case

$$\tfrac{1}{2}Li_b^2 = \tfrac{1}{2} \times 0.0121 \times 2.18^2 = 0.0288 \text{ joule.}$$

The measured output was 0.0193 joule or about 67 per cent. of the possible output.

On the other hand the input during make will of course be larger than the stored magnetic energy. In this case they are in the ratio of 0.0288 to 0.0417 or of the energy put in during wake, about 70 per cent. is stored as magnetic energy.

To recapitulate, by measuring for any coil the five quantities R, L, C, i_b, N_s/N_p, or the resistance, capacity, current at break and ratio of primary to secondary turns we can determine the four principal characteristics of the coil. Its secondary E.M.F. is given by

$$E_s = i_b \frac{N_s}{N_p} \sqrt{\frac{L}{C}},$$

its maximum output per spark by $\frac{1}{2}Li_b^2$, its frequency of oscillation by $n = 2\pi\sqrt{LC}$ and the maximum current at break by $i_{sm} = i_b \cdot N_p/N_s$.

A DIFFERENTIAL VOLUMENOMETER.

BY ALBERT P. CARMAN.

THE volumenometer or stereometer, invented by the French military engineer Say in 1797,[1] remains in some of its forms, the only instrument for determining the volumes of small irregular bodies and powders which cannot be immersed in a liquid. The fundamental principle of all volumenometers is the determination from Boyle's law of the volume of air imprisoned in a space V by observing the change of pressure corresponding to a given change of volume. The volume is changed in all forms by raising and lowering mercury in a graduated tube, the upper part of which is the vessel V. Then placing the body of volume x in the same space V, the new volume of air is found, and thus by subtraction, the volume x of the body is gotten. Most of the five or six volumenometers[2] which have been described and used in scientific work, differ simply in the methods of raising and lowering the mercury, that is, in the means of changing the volumes and measuring the change. In the volumenometer of Oberdeck we have an instrument which is distinct from previous forms in being differential. It was from a study of Oberdeck's instrument that the present work started. The first form described below was started simply as an improved form of Oberdeck's instrument, but it was found to have some new features both in theory and manipulation, and this led to experiments with modifications, as will be described.

Oberdeck's form is shown in Fig. 1; V and V' are two equal glass cups which can be closed by ground-glass plates; T and T' are graduated glass tubes fixed on a horizontal board and joined in a U at B. The U is connected by rubber tubing with a mercury

[1] Say, Ann. de Chem. et Phys. (1), XXIII., 1797.

[2] Regnault, Ann. de Chem. et Phys. (3), XIV., 1845. Kopp, Liebig's Ann., XXXV., 1840. Ruedorf, Wied. Ann., VI., 1879. Paalzow, Wied. Ann., XIII., 1881. Oberdeck, Wied. Ann., LXVII., 1899. Zehnder, Ann. d. Phys., X., 1903; XV., 1904.

reservoir *R*. By raising and lowering *R*, the volume of air imprisoned in *V* and *V'* can be changed. An essential feature of Oberdeck's instrument is that the pressures on the two sides are

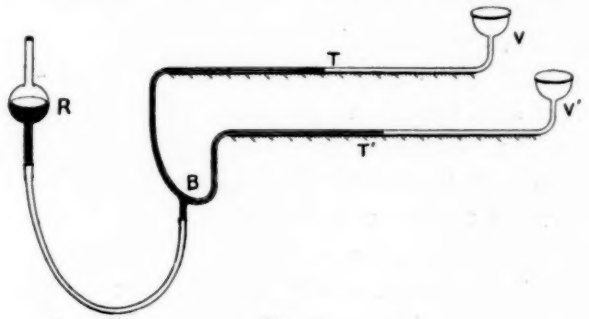

Fig. 1.

always equal, since the two tubes are on the same level. The increments of volume are read by the position of the mercury columns in the graduated tubes *T*. Some experiments showed that there must be considerable error in such readings owing to the lag of the mercury in horizontal tubes. It is also evident that the tubes *T* must be made long to get great sensitiveness, since the device of enlargements is not possible in horizontal tubes. To improve these features the instrument shown in Fig. 2 was made. *A* and *B* are vessels, each of about 5 c.c. capacity. A lid *L* can be clamped air-tight on *A*. *M* is a glass tube connecting *A* and *B*. *C* is a graduated glass tube. This tube is connected by rubber tubing with a mercury reservoir *R*. This reservoir can be raised and lowered so as to change the volume of the air imprisoned in *AMBC*. On the right side we have *A'M'B'C'R'*, a duplicate of the parts on the left side. The cups are connected by a sensitive manometer, containing

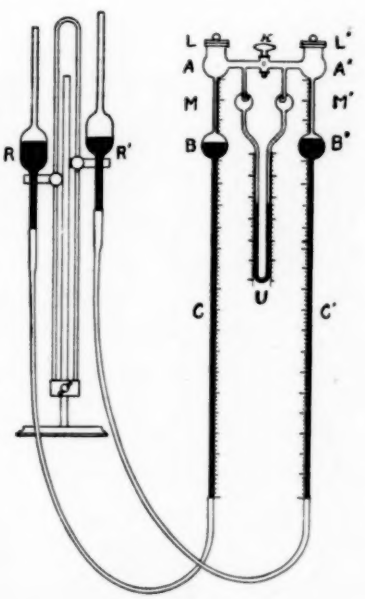

Fig. 2.

a light aniline liquid. A stopcock K allows the ready equalizing of the pressures on the two sides. The whole is mounted on a vertical board; on the same base is a vertical rod to which the platform carrying the mercury reservoirs is clamped. The reservoirs R and R' can be moved separately on rods carried by the common platform. In the first instrument made the vessels A and B were turned out of steel because they could be thus made exactly equal on the two sides. This equality is necessary in Oberdeck's method. Making these vessels of steel involved sealing them to glass and this caused more trouble and delay than anything in these experiments. The volumenometer proved a very sensitive method of detecting air leaks. An instrument has now been made all of glass with mercury sealed joints. This has solved the leak trouble.

The manipulation and theory are as follows: With the cups A open to the air the mercury is raised to the marks M and the vessels are then closed. Air is thus imprisoned at atmospheric pressure. The mercury reservoirs are lowered allowing the air volumes V and V' to expand by increments DV and DV' respectively. But during the operation the pressures on the two sides are kept equal by watching the manometer U. Letting p and p' represent the initial and final pressures, we have the relations

$$Vp = (V + DV)p' \quad \text{and} \quad V'p = (V' + DV')p'.$$

The mercury is next raised, restoring the volumes V. A body of volume x is now placed in A. The mercury is again lowered, keeping the pressures equal on the two sides, and giving the air in A, the previous increment DV. This will correspond to an increment DV'' on the right side for equality of pressure p''. We have then

$$(V - x)p = (Vx - x + DV)p'' \quad \text{and} \quad V'p = (V' + DV'')p''.$$

From the above we have

$$x = \frac{VDV'' - V'DV}{DV''}.$$

If the two volumes are equal, that is, $V = V'$, then

$$x = \frac{V(DV'' - DV')}{DV''}.$$

This is similar to Oberdeck's formula. It is believed that the vertical tubes and the sensitive method of determining equality of pressure on the two sides are an advance.

In working with the above volumenometer, an easier method of manipulation, using a simpler formula suggested itself. Starting as before, we have the relations

$$Vp = (V + DV)p' \quad \text{and} \quad V'p = (V' + DV'')p'.$$

After introducing the body of volume x, we increase the volumes keeping the pressures equal on the two sides, but giving to V' on the right side the previous increment DV'' and getting on the left the new increment DV'. Evidently the pressure is the same p' as above. We thus get

$$(V - x)p = (V + DV' - x)p' \quad \text{and} \quad V'p = (V' + DV'')p'.$$

Substituting for Vp and transposing we get

$$x = \frac{p'}{p - p'}(DV' - DV).$$

The difference of volume increments $(DV' - DV)$ is read off in terms of scale divisions of the tube C. The factor $p'/(p - p')$ can be determined directly from the pressures or it can be determined experimentally by placing a body of known volume in A and thus calculating it. This calibration has of course to be done at least once for each series on account of temperature and pressure changes, and the results are then read off direct in scale divisions of the tube. It is to be noted that no assumptions are made here of an equality of volume of the two sides. In getting the volume of turned brass cylinder, which could be accurately calipered, the results were 2.228 c.c. by the volumenometer and 2.210 c.c. by calipering. A steel ball was used for a standard. This shows the accuracy that has been attained ordinarily.

The above experiments were made in the summer and fall of 1906. In *Science Abstracts* for January, 1907, A. Lo Surdo's differential volumenometer is described. This has something of the appearance of the differential form described above, but it is in fact quite different in construction and manipulation.

UNIVERSITY OF ILLINOIS,
December, 1907.

THE VARIATION OF APPARENT CAPACITY OF A CONDENSER WITH THE TIME OF DISCHARGE AND THE VARIATION OF CAPACITY WITH FREQUENCY IN ALTERNATING CURRENT MEASUREMENTS.

BY B. V. HILL.

IT has been known that the impedance offered by a condenser to the passage of an alternating current depends upon the frequency of the current. The capacity is larger for low frequencies than for high, but the amount of this variation was not so generally known. A writer in the *American Telephone Journal* for September 29, 1906, stated that the paper condensers ordinarily used in telephone circuits often fall 50 per cent. below their rated capacity, and that the capacity varies greatly with the frequency of the current applied. The manufacturing companies only warrant an accuracy of 10 per cent. for apparatus of this kind, so a slight change of electrical constants with frequency is not a very serious matter — a dirty plug or a bent spring might make more difference in speech transmission than a small change in inductance or capacity in the circuit. It is of considerable importance, however, to be sure that a condenser rated at 2 M.F. really has that capacity and not merely half that amount. Several months ago Mr. Anthony Zeleny showed that the apparent capacity of a condenser, determined by the ballistic method, depended upon the period of the galvanometer, that is, upon the time during which the discharge of the condenser was to affect the needle or coil of the instrument. I decided to study the behavior of several condensers both with reference to their straight discharge and their capacity when a part of an alternating current.

For this purpose there were at hand six condensers of different kinds. They were, as designated below: (1) a Leeds and Northrup standard mica condenser of 1 M.F. capacity; (2) a Queen and

Company's paper condenser of 1 M.F. capacity; (3) and (4) telephone condensers of one firm rated at 2 M.F. capacity; (5) a telephone condenser of another firm rated at 2 M.F. capacity, and a sixth made by the same firm as (3) and (4) but of so low insulation resistance that no results could be obtained with it.

The straight discharge was first studied, the method being to charge the condenser for 20 seconds and then, after a period of insulation of about .07 second to connect to the ballistic galvanometer for times varying from .0001 second to 11 seconds, the quarter period of the galvanometer. To measure the times of connection with the galvanometer, two pairs of keys, a make- and a break-circuit key in each pair, were made and mounted on heavy maple blocks faced with ebonite. These keys were very similar to those ordinarily used in such experiments and described in the texts on electrical measurements, and so need not be described in detail here. Instead of the heavy pendulum ordinarily employed to release the keys, I mounted a 7/8″ cold drawn steel rod in a vertical position near a strip, also vertical, upon which the keys were mounted. A weight sliding upon this rod struck the triggers and released the keys. The positions of the triggers were determined by means of a fine cathetometer. The friction of the weight upon the rod was first carefully determined and taken into account in computing the time between the setting off of the two triggers. The break-circuit key on each block was provided with a screw by which it could be moved with reference to the make-circuit key by very small amounts. The zero setting was approximated by finding a place where there was no throw of the galvanometer when the weight was released, but, if the breaking key was moved downward by the smallest possible amount, a throw was observed. Settings agreeing to 1/6,000 of a second could thus be made. This does not represent the accuracy of the apparatus but the smallest distance through which the fingers could turn the screw. A series of readings was now taken. The condenser was first allowed to charge for 20 seconds. The weight was then released and, in its fall, broke the charging circuit, and, after .067 second struck the triggers, first connecting and then disconnecting the condenser and the galvanometer. The times of connection

were plotted as abscissæ and the corresponding throws of the galvanometer as ordinates. The curve thus obtained was produced back across the axis of times and the point where it cut this line was taken as the true zero point.

The galvanometer was a Leeds and Northrup type P d'Arsonval. It had a resistance of 127 ohms and an inductance of .0014 Henry. The coil was loaded with two small bullets so that its period was 43.08 seconds.

The behavior of the four condensers which were tested is shown, for times up to .01 second in the figures. Curve I. in the first figure is the theoretical discharge curve for the capacity of one microfarad and the corresponding curve of the second figure is the theoretical curve for the capacity of two microfarads. Curves II. and III., Fig. 1, refer to the Leeds and Northrup mica condenser and the Queen paper condenser respectively. With the former the effect

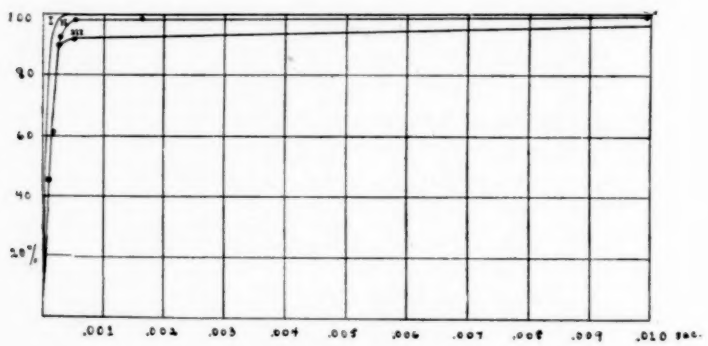

Fig. 1.

of absorption is over in a very short time but, even here, the galvanometer must have a period of at least 5 seconds if the complete charge is to be liberated. Using a Kemp's key with the ballistic galvanometer, the calculated capacity of this condenser was 1.0328 M.F. With the paper condenser the throw, after 8 seconds connection, is still 2 per cent. below that obtained with the Kemp's key and the capacity calculated from this full throw was 1.0519. These values may seem absurdly high, especially for so fine a piece of apparatus as the mica condenser; but these same condensers have been used repeatedly for the determination of the earth's horizontal

component H and the value of this constant was uniformly lower than that obtained by Gauss' method, by an amount corresponding to the excess of capacity just shown in the condensers. From the results obtained in the alternating current measurements, the mica condenser appears to be very accurate and the fault is with the ballistic method of measuring the capacity.

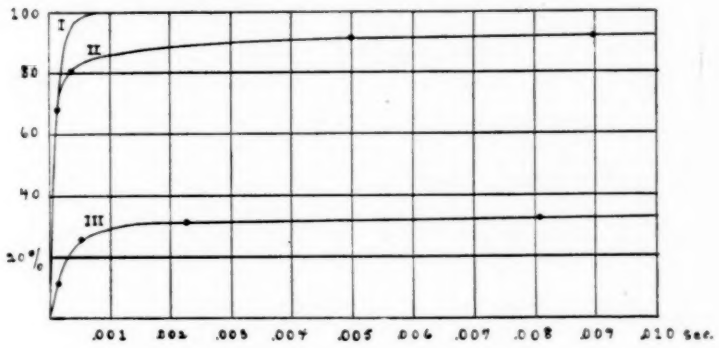

Fig. 2.

Curves II. and III. of the second figure refer to condensers (3) and (4), both being 2 M.F. paper telephone condensers. The capacity of a given condenser of this type increases, of course, with the amount of pressure applied in the making, while the insulation resistance decreases. That designated as (3) is of high resistance and is what they rate as a "hard" condenser; (4) is a medium, while the sixth one referred to in the beginning of this paper as giving no proper result at all, is called "soft." No. 3 appears to be a very good condenser of its class but (4) has parted with only about half of its charge at the end of .01 and is found to continue discharging for two minutes or more.

For determining the capacity of the condensers by means of alternating currents, it seemed best to use some method of comparing their impedance with that caused by a self-induction; for, even if there should be a small error in the value assumed for the latter, the data sought, namely, the variation of capacity with frequency, will be as valuable as ever. The coil used as a standard of inductance was of No. 23 copper wire, insulated with silk and wound upon a marble cylinder. Its inductance was .011316. For such a coil the

effect of frequency upon resistance and inductance, for frequencies up to 3,000, is too small to be taken into account here. After trying several of the standard arrangements for comparing a capacity to an inductance a modification of Anderson's method was used. This involved the fewest assumptions and will allow a large number of measurements to be made in rapid succession. The arrangement shown in Fig. 3 is the common one except that, by means of a double key, each half having six points, a battery and a galvanometer, or a source of alternating current and a receiver, may be connected to the points of the bridge by merely throwing the lever. Leaving the key in the normal position, as shown in the figure, one may adjust the arms of the bridge for a direct-current balance and then, throwing the key into the other position, adjust the resistance r till the sound in the receiver is a minimum. The capacity is now given by the equation

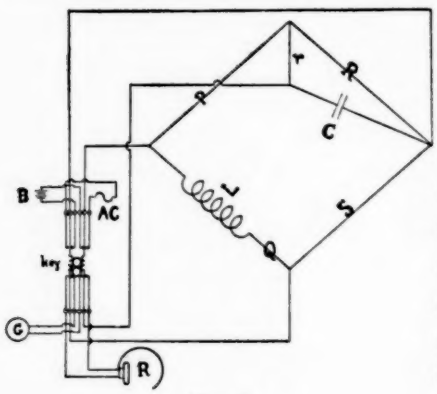

Fig. 3.

$$C = \frac{L}{r(Q+S)+PS}.$$

To secure alternating current of varying frequencies up to 3,000, which was sufficiently high to represent ordinary voice currents, a small inductor-alternator was constructed. It consisted of an ebonite disk having about its circumference 60 soft iron, cylindrical inductors .25 in. in diameter, rotating between the poles of the field magnet. The armature coils were wound upon these poles. As the whole effect found was small, no account was taken of wave-form. For the very low frequencies (16 and 66) the currents from a pole-changer were used.

The results thus obtained are shown in the following table:

Numbers 1, 2 and 3 increase very slightly in capacity between 16 (or 66) and 125 cycles per second. The other two do not show this increase but all fall off regularly up to the highest frequencies

Condenser.	Capacity (Ball).	Frequency.	Capacity.	Per Cent. Loss.
1	1.0328	66	0.9961	
		125	0.9970	
		1,450	0.9951	
		2,945	0.9931	0.4
2	1.0519	16	0.9985	
		125	1.0005	
		1,360	0.9965	
		3,020	0.9901	0.8
3	2.170	16	2.0328	
		125	2.0367	
		990	2.0245	
		3,040	2.0064	1.49
4	2.170	16	1.8519	
		125	1.8519	
		1,350	1.8379	
		3,050	1.8046	2.55
5		16	2.3448	
		125	2.3420	
		1,350	2.3231	
		3,000	2.3066	1.10

tried. The worst, however, decreases but 2.55 per cent., which is an amount too small to be of account in considering their effect upon speech currents. As the frequency rises, the minimum of sound in the receiver becomes less marked in all cases. With the poorer paper condensers, this is true to such an extent that no very exact adjustments could be made, but the capacity of these condensers was somewhat indefinite.

It will be seen by comparing the discharge curves of the several condensers with their alternating-current capacities as given in the table, that the latter cannot be inferred from the former. Taking condenser No. 4, as an example, it will be seen that, up to .01 second, the capacity is only about 37 per cent. of that obtained when the condenser was allowed to discharge for the whole quarter period of the galvanometer. With an alternating current, however, in which the period of charge — that is, the quarter-period of the complete oscillation — was only .00008 second, the capacity was still 80 per cent. of that obtained by the ballistic method and the complete quarter-period of the galvanometer.

CHICAGO,
December 30, 1907.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE FORTIETH MEETING.

A REGULAR meeting of the Physical Society was held at Columbia University, New York City, on Saturday, February 29, 1908.

At the afternoon session, President Edward L. Nichols delivered an address on "Theories of the Color of the Sky."

The following additional papers were presented:

A Study of Anomalous Dispersion by Means of Channeled Spectra. S. R. WILLIAMS.

A New Form of Interferometer. C. W. CHAMBERLAIN.

Some Properties of Light of Extremely Short Wave-length. THEODORE LYMAN.

The Variation of the Thermomagnetic Effect in Soft Iron with the Strength of the Magnetic Field and with the Temperature Gradient. L. L. CAMPBELL.

The Kinetic Energy of the Negative Ions from Hot Metals. O. W. RICHARDSON and F. C. BROWN.

A Development of Wiedemann's Theory of Luminescence, with especial reference to the Decay of Phosphorescence. ERNEST MERRITT.

A Curious Ionization Effect in Vacuum Tubes. H. A. PERKINS.

Exhibition of a New Form of Colorimeter. F. E. IVES.

A Sine-wave Electrical Oscillator of the Organ Pipe Type (with demonstration). FREDERICK K. VREELAND.

The Origin and Life of Radium. BERTRAM B. BOLTWOOD.

Heat Developed in a Mass of Thorium Oxide, due to its Radioactivity. GEORGE B. PEGRAM and HAROLD W. WEBB.

Contact Electromotive Force and Cohesion. (By title.) FERNANDO SANFORD.

Notes on Superheated Steam. HARVEY N. DAVIS.

Adjourned at 5 P. M.

ERNEST MERRITT,
Secretary.

Notes on Superheated Steam.[1]

By Harvey N. Davis.

1. *On its Specific Heat at Constant Pressure.* — This note discusses the available data, and in particular the papers of Knoblauch and Jacob, of the Technische Hochschule in Munich, and of Thomas, of Sibley College, Cornell, and leads to the conclusion that in the two respects in which these papers are in serious disagreement, the results of Knoblauch and Jacob are to be preferred.

2. *On its Total Heat.* — During the last ten years several careful sets of "wire-drawing" experiments have been undertaken in the hope of computing C_p from them by means of Regnault's values for the total heat of saturated steam; but, as was pointed out by the experimenters themselves, this procedure is extremely sensitive to errors in Regnault's table, so that the resulting values of C_p are not to be compared with those since obtained by direct calorimetry. It is the purpose of this note to point out that the work of Knoblauch and Jacob makes it possible to reverse this process so as to obtain, not indeed absolute values, but a knowledge of the variation of the total heat with temperature in the range from 100° C. to 180° C. which is more trustworthy than anything previously reported. The original data of Grindley, Peake and Griessmann have been reduced from this point of view and the resulting curves show an unexpectedly satisfactory agreement. The average temperature coefficient from 100° to 180° is very nearly the well known 0.305 of Regnault, but all three lines show an unmistakable curvature, being concave on the side toward the temperature axis. A similar result has been deduced by Linde from the measurements by Knoblauch, Linde and Klebe of the specific volume of saturated steam. All three sets of observations are well represented by the empirical formula

$$\lambda = \lambda_{100} + 0.3745(t - 100) - 0.000990(t - 100)^2.$$

Since all three curves are well defined near 100° (and one of them, indeed, down to 60°) an attempt was made to connect up with the absolute determinations at these lower temperatures so as to get at absolute values throughout the range under consideration, but the outstanding discrepancy of nearly two calories between Regnault and Henning at 100° makes this somewhat uncertain. The only previous determinations above 100° are the experiments of Regnault and the computations of Linde.

3. *On its Joule-Thomson Effect.* — Buckingham in a very interesting paper recently published by the Bureau of Standards, follows several

[1] Abstract of a paper read before the American Physical Society, in New York, February 29, 1908.

earlier writers in basing an evaluation of the absolute thermodynamic scale upon the very reasonable assumption that the law of corresponding states applies to the Joule-Thomson effect in air, oxygen, nitrogen and hydrogen. He further assumes that it applies also to carbon dioxide, but the dependence of his thermometric conclusions upon this second assumption is probably so indirect that our confidence in them ought not to be much shaken even if it turns out that CO_2 does not belong in the same group with the diatomic gases, provided that the law of corresponding states holds for them. It should be remembered, however, that this law has never been proved experimentally for any pair of them whatever, so that, although we accept Buckingham's results with confidence, nevertheless, an experimental investigation of the whole subject, and in particular of the possibility of classifying all sorts of substances on the basis of a law of corresponding states, is much to be desired.

It is the purpose of this note to make a small contribution in this field by evaluating the Joule-Thomson effect in H_2O and comparing it with that in CO_2. The comparison is interesting as being the first of its kind, for carbon dioxide is the only gas for which we have hitherto had anything more than isolated values of the Joule-Thomson effect. For the evaluation for water vapor there are available, (1) the "wire-drawing" experiments already mentioned, which give about fifty direct experimental determinations, (2) some wire-drawing data at very high temperatures, hitherto unpublished, which have recently been generously put at my disposal by Mr. A. R. Dodge of the General Electric Company, and (3) the curves of constant "total heat content" that can be computed from the results in the first two of these notes. The resulting "reduced" values, when plotted, form a band which, although wider than might be wished, is quite definite enough for the present purpose, and which forms a surprisingly good continuation of a smooth curve through Kester's values for carbon dioxide. This confirmation of the law of corresponding states is the more striking because water is so anomalous in other ways that even a complete failure of the law would not have been surprising. *A fortiore* is it to be expected that the thermometric gases will satisfy the law.

I have examined all of the available data on steam for a possible variation of the Joule-Thomson effect with pressure without success. If it exists, it is within the limit of error of the observations.

I hope that these notes will be published in the proceedings of the American Academy of Arts and Sciences.

THE JEFFERSON PHYSICAL LABORATORY,
CAMBRIDGE, MASS.

The Kinetic Energy of the Negative Ions from Hot Metals.

By O. W. Richardson and F. C. Brown.

THE kinetic energy of the ions has been measured by observing the rate at which the potential of one of a pair of parallel plates charged up when a small area (about one square millimeter) of the opposite plate was occupied by heated platinum foil. The hot platinum and the plate surrounding it were maintained at zero potential. If the particles are shot off from the foil with a velocity component u perpendicular to the plates they will only reach the upper plate provided $\frac{1}{2}mu^2 > Ve$ where m is their mass and e their electric charge, V being the difference of potential between the plates. The method thus measures the number of particles whose component of energy in the direction perpendicular to the plates (if we may be allowed to use this expression) is greater than a given value. It is evident that the results give the average value of this part of the energy and also the way in which it is distributed among the different particles emitted.

On the assumption that the ions shot off from the hot metal behave like the molecules of a gas it can be shown that the number n which reach the upper plate against a potential V is given by $n = n_0 e^{-\nu Ve/R\theta}$ where n_0 is the number which reach the upper plate when $V = 0$, ν is the number of molecules in 1 c.c. of a gas at normal temperature and pressure, θ is the absolute temperature and R is the constant in the gas equation $pv = R\theta$. This formula is equivalent to $\log i - \log i_0 = -\nu Ve/R\theta$ where i and i_0 are the currents corresponding to n and n_0. This has been tested by plotting the experimental values of $\log i$ and V and the linear relation demanded by theory has been accurately verified. The inclination of this line is equal to $\nu e/R\theta$. Now νe is the quantity of electricity required to set free half a cubic centimeter of hydrogen at 0° C. and 760 m.m. pressure by electrolysis. Substituting the value of this constant and that of θ the temperature of the wire we find the value of R, the gas constant for the negative electrons. Different experiments have given values ranging from 3.5×10^3 to 4.05×10^3 as compared with the standard value 3.7×10^3.

We are thus led to the conclusion that the kinetic energy of the negative electrons emitted by hot metals is the same as that of the translational kinetic energy of the molecules of a gas at the same temperature as that of the metal.

If we admit the truth of the atomic theory and the correctness of the accepted values for the mass of an atom this investigation furnishes what

[1] Abstract of a paper read before the American Physical Society, in New York, February 29, 1908.

we believe to be the first direct test of certain theorems in the kinetic theory of gases relating to the way in which the velocities are distributed among the different atoms.

It is also suggested that a method based on these principles will ultimately be of value, on account of its practical simplicity and theoretical directness, as a standard of thermometry for very high temperatures.

PRINCETON, N. J.,
February 28, 1908.

HEAT DEVELOPED IN MASS OF THORIUM OXIDE, DUE TO ITS RADIOACTIVITY.[1]

BY GEO. B. PEGRAM AND HAROLD W. WEBB.

ABOUT four kilograms of thorium oxide were placed in a spherical Dewar bulb, and this, with a metal cylinder for protection, was put in an ice-bath.

Thermoelements with one set of junctions in the thorium oxide, the other set at the temperature of the ice, showed that the mass of thorium oxide maintained a higher temperature than the ice-bath. Heat was then introduced at a known rate into the mass of thorium oxide, by sending a small current through wires laid in the thorium oxide. Comparing the temperature elevation due to the radioactivity of the thorium oxide alone with the temperature elevation maintained when heat was being introduced at a known rate by the current, the thorium oxide used was found to be liberating heat at the rate of $.96 \times 10^{-5}$ calories per hour per gram, or about $\frac{1}{10}$ erg per second per gram. From this value the heat developed by pure thorium oxide in its equilibrium state was calculated to be 2×10^{-5} calories per hour per gram. Measured by the total energy liberated, thorium oxide is then about a three millionth part as radioactive as radium bromide.

CONTACT ELECTROMOTIVE FORCE AND COHESION.[1]

BY FERNANDO SANFORD.

IN the consideration of the phenomenon of contact electromotive force in metals the writer has been led to the conclusion that it is definitely related to the specific inductive capacity, so that a metal having a higher specific inductive capacity will become positively electrified when in contact with a metal having a lower specific inductive capacity. This law has already been shown by Coehn (Annalen, 64, 217) to apply to non-metallic bodies.

[1] Abstract of a paper read before the American Physical Society, in New York, February 29, 1908.

It also seems probable that cohesion, like chemical affinity, is due to electric attractions, and since the electric attraction between two charged bodies is inversely as the specific inductive capacity of the intervening medium, the attractions between the molecules of bodies should vary according to the same law.

The above assumptions lead to the conclusion that the voltaic series should be inversely as the cohesion series.

Lacking direct data on the cohesion of metals, I have attempted to test the above hypothesis by relating the voltaic series to the melting points, the expansion coefficients and the hardness of the metals. The recent valuable work of Professor T. W. Richards and his students on compressibility has given still better data for estimating the cohesion between the molecules of metals. I would accordingly beg leave to submit to the Physical Society the following table, which seems to me to suggest more than an accidental relation between electromotive force and cohesion.

Voltaic Series.	Compressibility $\times 10^6$.	Melting Point.	Expansion Coefficient.	Hardness.
Cæsium	61	26.5		.2
Rubidium	40	38.5		.3
Potassium	31.5	58		.5
Sodium	15.5	95	.000072	.4
Lithium	8.8	180		.6
Zinc	1.5	419	.000029	2 5
Lead	2.2	330	.000028	1.5
Tin	1.67	230	.000022	1.8
Iron	.38	1,500	.000012	4.5
Silver	.82	950	.000019	2.5–3
Copper	.54	1,054	.000017	2.5–3
Gold	.47	1,035	.000015	2.5–3
Platinum	.21	1,780	.000009	4.3
Carbon	.5	unmelted	.000005	10

As will be seen, iron is the most marked exception in the list. Its other properties would make it seem to belong just above platinum in the voltaic series. It is known that iron may exist in two allotropic conditions, but the temperature of its non-magnetic form is much above that of the measurements under consideration. Its thermo-electric curve also indicates that its position in the voltaic series would be very different at either higher or lower temperatures.

Zinc also seems to be out of place in the series as indicated by all the properties considered except its expansion coefficient. Its other properties would indicate that it belongs between lead or tin and silver in the voltaic series. It may, perhaps, be questioned whether the zinc which

has been used in contact electromotive force determinations has been carefully purified.

The data given in the above table has been taken mostly from Landolt and Boernstein's tables except the compressibility, which is taken from Professor Richards's article in Zeit. phys. Chem., LXI., 196, December, 1907.

STANFORD UNIVERSITY,
February 14, 1908.

SOME PROPERTIES OF LIGHT OF EXTREMELY SHORT WAVE-LENGTH.[1]

BY THEODORE LYMAN.

SCHUMANN has investigated the absorption of some gases in the region of the spectrum lying on the more refrangible side of λ 1,850. The present paper is chiefly concerned with a repetition and extension of this work. The writer has employed a vacuum grating spectroscope; the superiority of this instrument for purposes of exact measurement, over the prism apparatus of the earlier investigator, has led to some new and important results. A detailed report of these results will be found in the Astrophysical Journal for March, 1908; they may however be briefly stated as follows:

Oxygen, in thicknesses of 1 cm., absorbs strongly in the region more refrangible than λ 1,850, but contrary to the results obtained by Schumann, the absorption is in the form of a band. Hydrogen, argon and helium in thicknesses of about 1 cm. are transparent. Nitrogen in the same thickness absorbs very slightly. Carbon monoxide and carbon dioxide absorb quite energetically, but their action is unlike that of oxygen, since in place of one broad region of absorption, they give rise to eight or more narrow bands. It appears that the behavior of the air is to be ascribed to the action of the oxygen and nitrogen which it contains. The addition of ozone either to air or to oxygen does not seem to affect the absorption of these gases in the region under consideration. The factors, therefore, which set the limit to the extent of the solar spectrum may be different from those which operate in the region of light more refrangible than λ 2,000. For between λ 2,000 and λ 3,000 ozone is known to absorb strongly.

It seems probable that the absorption of oxygen in the region below λ 1,850, which so strongly distinguishes it from the other elementary gases, may be ascribed in part to photo-chemical action. It has long been known that ultra-violet light produces ozone peroxide. The writer has now shown that this action increases rapidly with decrease

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in wave-length below λ 1,850. So pronounced is this effect that the difference in transparency between quartz and fluorite is readily detected by observing the relative discolorations produced in starch-potassium-iodide test paper when the substances under investigation are interposed successively between the paper and the source of light. The great absorption of a layer of air one millimeter thick may also be easily demonstrated by the same simple means.

Although the investigation has been confined to a single gas path about 1 cm. in thickness, the effect of change of pressure on the absorption has been studied. In the case of oxygen increase in pressure produces an unsymmetrical broadening of the band. This fact taken in conjunction with the experiments on the formation of ozone or hydrogen peroxide are interesting in connection with a theory due to Larmor, which connects the unsymmetrical shifting of the limits of an absorption band with the formation of molecular complexes.

The absorption of oxygen for wave-lengths shorter than λ 1,850 is a phenomenon of practical importance and one that characterizes the region discovered by Schumann. It is not, however, the only property which renders this little known part of the spectrum interesting. The increase in photo-chemical effect, in photo-electric action and in volume ionization with increase in the period of the exciting light, together with the striking peculiarities of the spectra of various substances in this region, all combine to indicate that the study of vibrations of very high period may be expected to yield results bearing on some of the relations between ether and matter.

JEFFERSON PHYSICAL LABORATORY,
February 28, 1908.

THE ORIGIN AND LIFE OF RADIUM.

BY BERTRAM B. BOLTWOOD.

A PRELIMINARY notice has already been published (Nature, October 26, November 10, 1907; Am. Jour. Sci., XXIV., p. 370, 1907) of the separation and identification of a new radioactive element which is present in uranium minerals. Further quantities of this element, to which the name "ionium" has been given, have been obtained in the form of a more highly concentrated preparation having an activity about 2,000 times that of an equal weight of pure uranium. With this preparation, which is essentially free from other radioactive substances, it has been possible by the Bragg method to determine the range in air of the α particles emitted by ionium. The range is found to be approxi-

[1] Abstract of a paper read before the American Physical Society, in New York, February 29, 1908.

mately 2.80 cm. It is probable that the disintegration of ionium is also accompanied by the expulsion of β particles. Experiments on the relative activities of the different radioactive constituents of a uranium mineral indicate that the values are approximately the following, the activity of the uranium being taken as unity:

Element.	Activity.
Uranium,	1.00
Ionium,	0.35
Radium,	0.45
Radium emanation,	0.62
Radium *A*,	0.54
Radium *C*,	0.91
Radium *F* (polonium),	0.49
Actinium products,	0.36
Sum of activities,	4.72

This value for the sum of the activities is in good agreement with the value obtained directly by measurements of uranium minerals which indicate that the minerals are 4.69 times as active as the uranium which they contain.

The relative activities found for the radium and ionium are in good agreement with the relative ranges in air of the α particles emitted by these elements which are 3.5 cm. and 2.8 cm. respectively.

Experiments on the growth of radium in solutions of ionium, prepared by various methods from definite quantities of minerals containing known amounts of uranium and radium, indicate that the amount of radium produced in one year by the ionium associated with one gram of radium in minerals is approximately 3.45×10^{-11} grams. This gives the time required for a given quantity of radium to be half transformed into other products (the half-value period) as 2,000 years, with the probability that this differs by not more than 100 years from the true period. Details of these experiments will be published shortly.

A STUDY OF ANOMALOUS DISPERSION BY MEANS OF CHANNELED SPECTRA.[1]

BY S. R. WILLIAMS.

SOME years ago, a method was described in the PHYSICAL REVIEW for determining the refractive indices of transparent substances by means of channeled spectra. In the present work this method is applied to the study of anomalous dispersion in highly absorbing media.

Solutions of cyanin, litmus, eosin, naphthalin red and fuchsin in alcohol

[1] Abstract of a paper read before the American Physical Society, in New York, February 29, 1908.

were first investigated. The concentrations were those used by Bates in studying magnetic rotatory dispersion. Mr. Bates found no anomalous rotatory dispersion in these solutions, and to corroborate his work asked that these solutions be tested for anomalous dispersion since the dispersion formula indicate that a substance having anomalous rotatory dispersion must also show anomalous dispersion. The results of this investigation substantiate Mr. Bates's work.

The concentration of the fuchsin solution was then increased to two, three and four times that of the original solution, and a study made of these solutions. Even at these concentrations no anomalous dispersion was found although the refinement of the method was such that, for a change in index in the fifth place beyond the decimal point, a shift in the bands would occur, visible to the naked eye. This is a sensibility not to be attained by prismatic methods in highly absorbing media.

A Curious Ionization Effect in a Vacuum Tube.[1]

By H. A. Perkins.

THE following curious effect was discovered in the course of some experiments with the electrodeless discharge. A tube about 20 cm. long, and 1 cm. diameter, having electrodes sealed into its two ends was exhausted to what is popularly known as a "white vacuum." This tube was surrounded by a narrow metal ring of a diameter slightly larger than the tube, and the ring was connected to one terminal of a transformer, the other terminal of the transformer being earthed. When the transformer was giving an E.M.F. of 550 volts or over and a frequency of 60 cycles, the tube showed a marked luminescence and a direct current flowed through a galvanometer connected in series with the tube. The terminal of the tube nearest the exciting ring was always positive. The current was very small when the ring was near the end of the tube, but increased rapidly as the ring neared the center. At the center it reversed in direction coming rapidly to another maximum just beyond that point, and then fell rapidly away as the ring neared the far end of the tube.

A battery inserted in series with the tube could greatly increase these currents or reverse them according to the direction of its E.M.F. and this effect as well as the preceding seemed to indicate much greater ionization when the ring was near the center of the tube.

An electrostatic voltmeter connected to the terminals of the tube indicated its largest values when the ring was nearest one end; these values diminishing to 0 at the center; but the time required for it to reach a steady reading was very great when the ring was near one end, and the

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larger voltages were obtained, thus apparently indicating a much feebler ionization in that position.

To account for these phenomena, a theory, depending on the difference of speed of the ions, has been partly worked out, which accounts satisfactorily for the polarity of the tube, and gives an equation roughly approximating to the current curve obtained.

THE VARIATION OF THE THERMOMAGNETIC EFFECT IN SOFT IRON WITH THE STRENGTH OF THE MAGNETIC FIELD AND WITH THE TEMPERATURE GRADIENT.[1]

BY L. L. CAMPBELL.

THE transversal thermomagnetic effect was first determined in the soft iron under a constant gradient of temperature and a varying strength of magnetic field. The temperature gradient was about 18° C. per. cm., and the magnetic field varied from about 6,000 to 12,000 maxwells per cm. The effect increased with, but was not a linear function of, the field strength. The thermomagnetic rotation coefficient Q increased with the field for awhile, and then gradually decreased. For one iron plate, Q had an initial value of 0.00147, a maximum of 0.00150, and a final value of 0.00148. At the higher limit of the field strength, the effect did not seem to approach a saturation value.

The second determination was made under a constant field strength of about 9,000 maxwells, and the temperature gradient varied from 15° C. to about 65° C. per cm. The effect here also increased with the gradient, but was not a linear function of it. For the same iron plate as mentioned above, Q increased from 0.00146 to 0.00161, and then fell off to 0.00134. At the highest gradient employed, the effect seemed to approach a saturation value.

In the light of the above determinations, it would seem well, in quoting values of the coefficient Q for metals, always to state definitely the field strength and temperature gradient under which the determinations were made.

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[1] Abstract of a paper read before the American Physical Society, in New York, February 29, 1908.